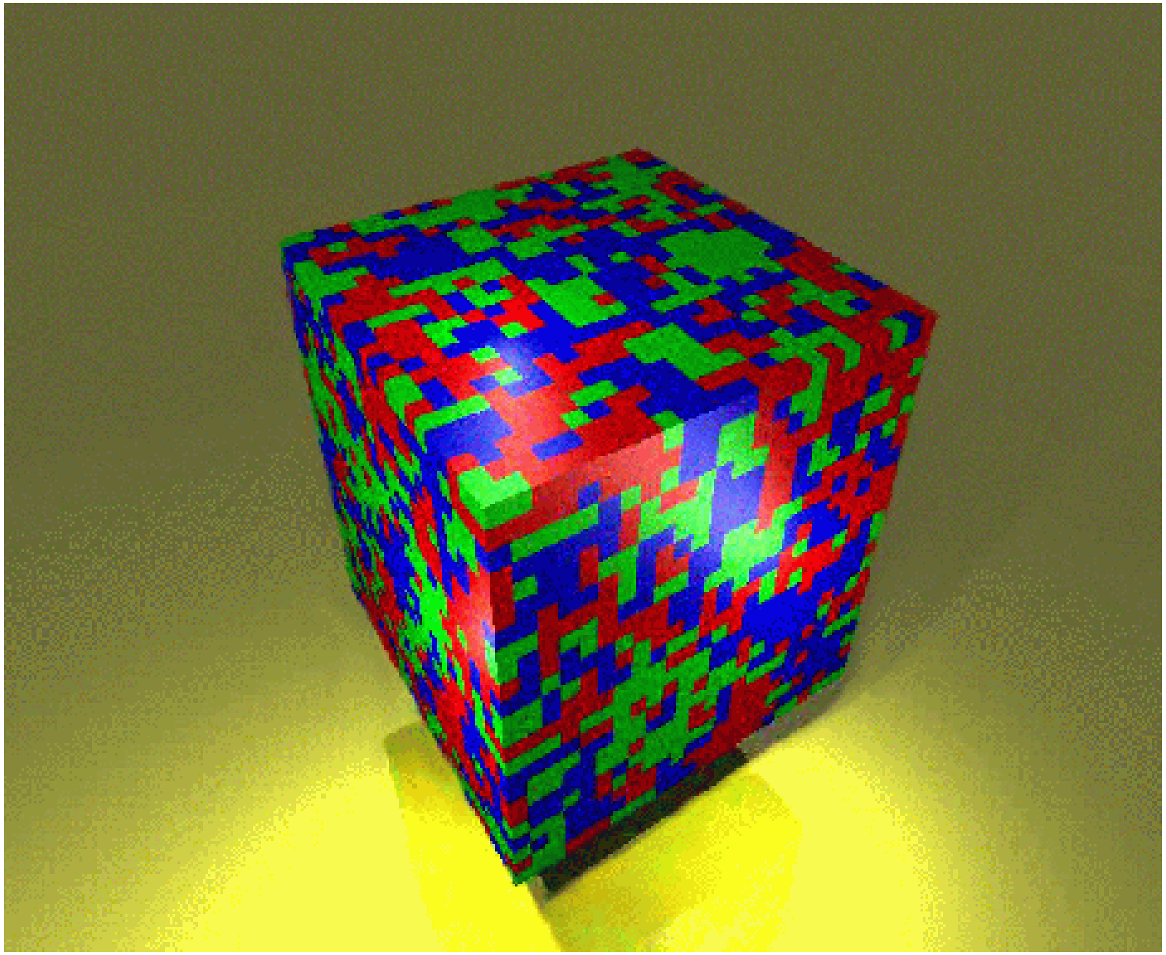
Lattice Gauge Theory

Frithjof Karsch, BNL&Bielefeld

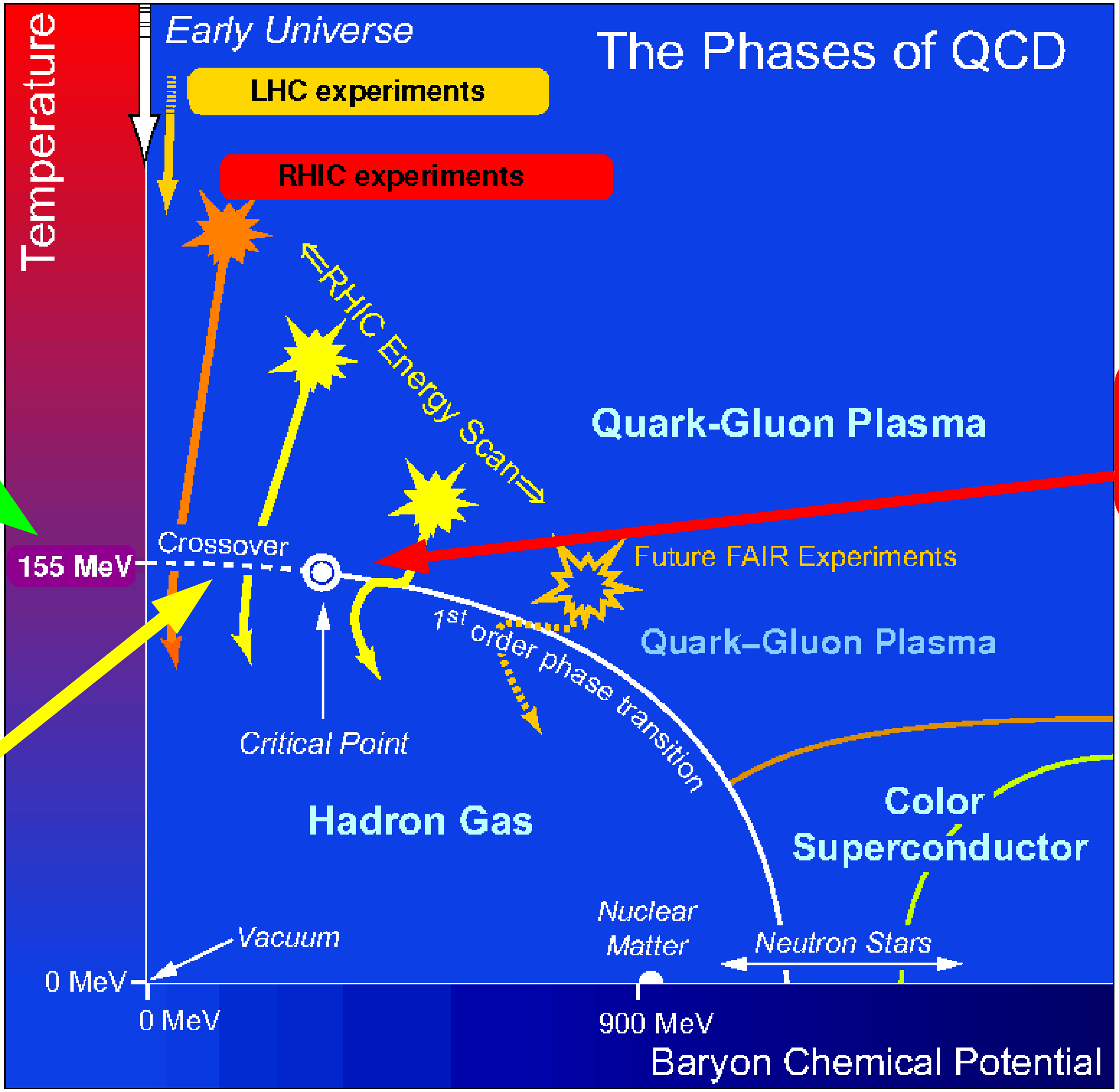


OUTLINE:

- lattice regularization and continuum limit
- QCD close to the chiral limit, O(N) scaling, phase diagram
- finite density QCD moments of charge fluctuations as probe for for proximity to criticality

Phases of strongly interacting matter

crossover temperature

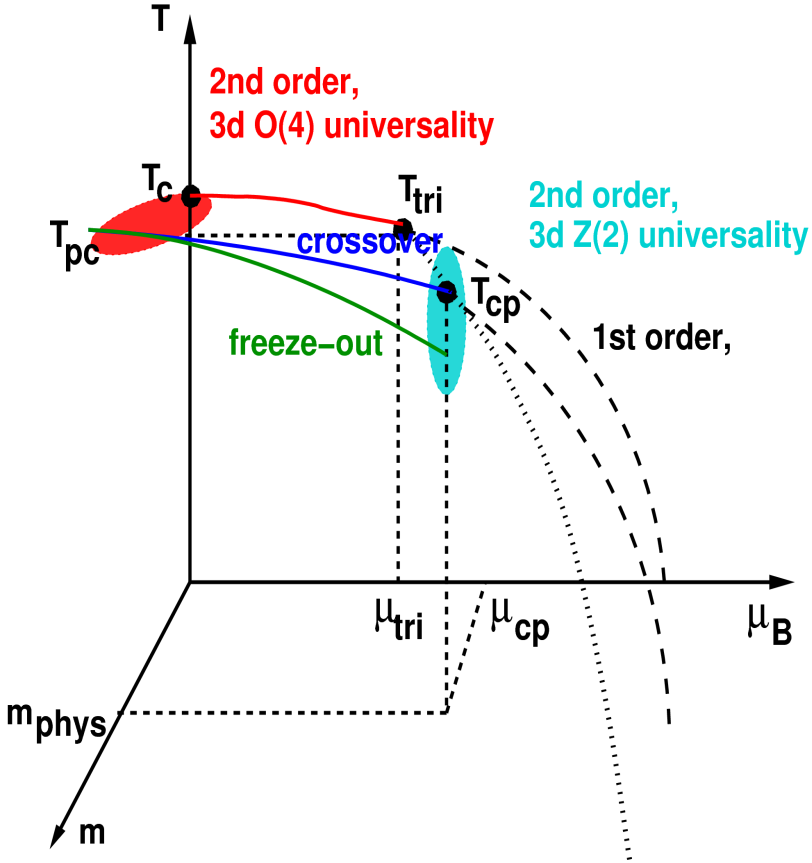


existence of a critical point

curvature of the crossover line

Phase diagram for $\mu_B \geq 0, \ m_q > 0$

Does freeze-out occur close to a critical point?



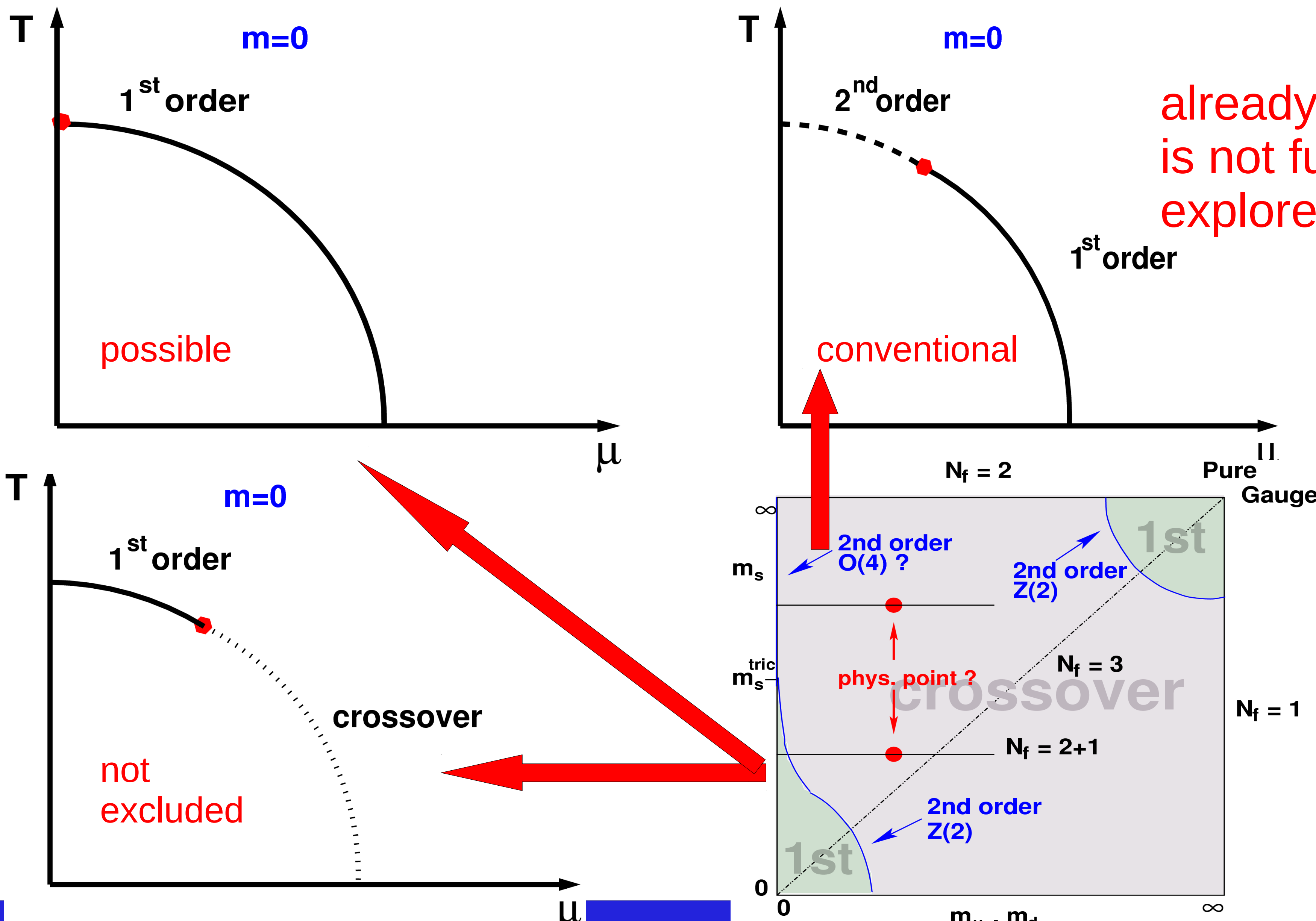
- critical line at m_q=0

$$\frac{T_c(\mu_B)}{T_c} = 1 - \kappa_B \left(\frac{\mu_B}{T}\right)^2 - \mathcal{O}(\mu_B^4)$$

- crossover line
  physics on crossover line controlled by universal scaling relations ?

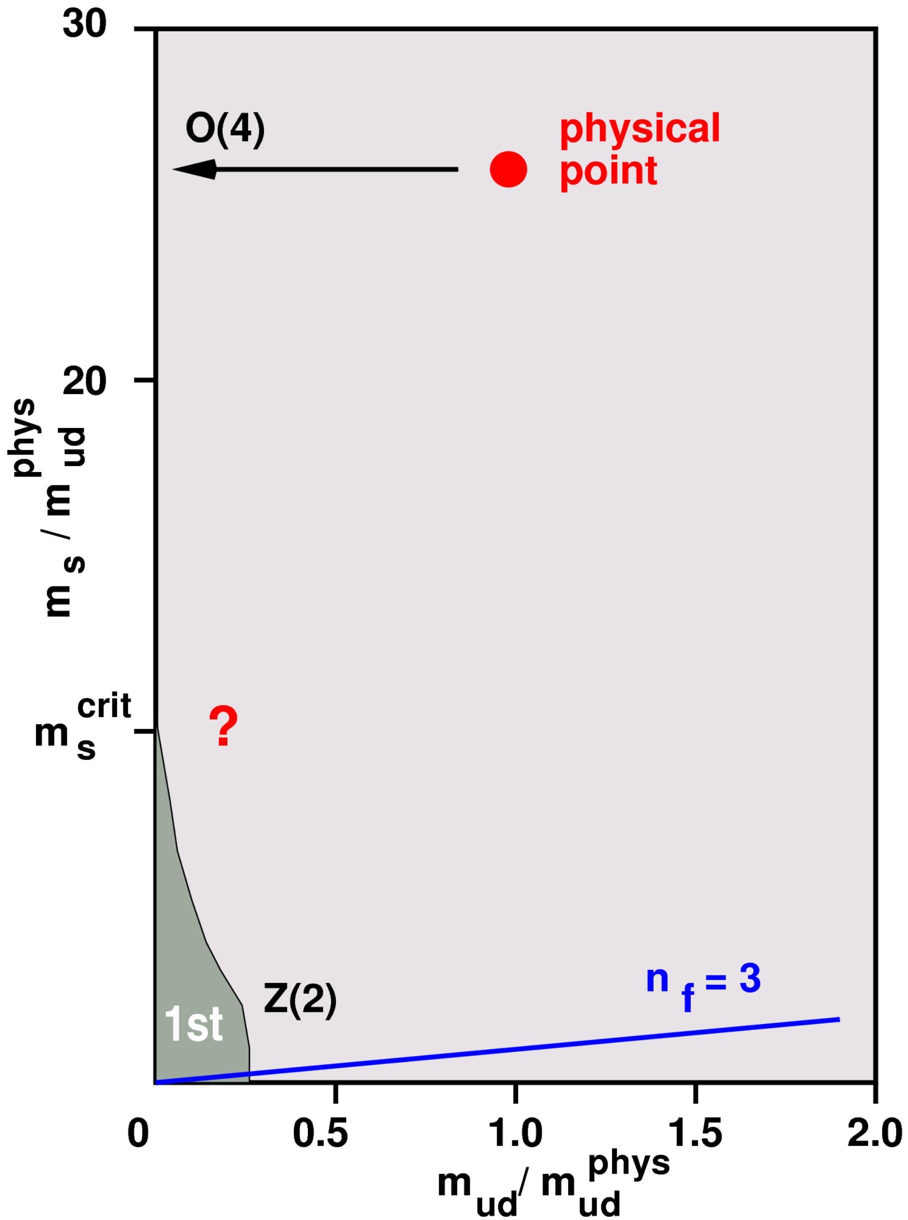
- freeze-out line
  Is the crossover line related to the experimentally determined freeze-out curve?

Critical behavior in hot and dense matter
QCD phase diagram & chiral limit



already $\mu_B = 0$ is not fully explored

Phase diagram for $\mu_B = 0$



- drawn to scale

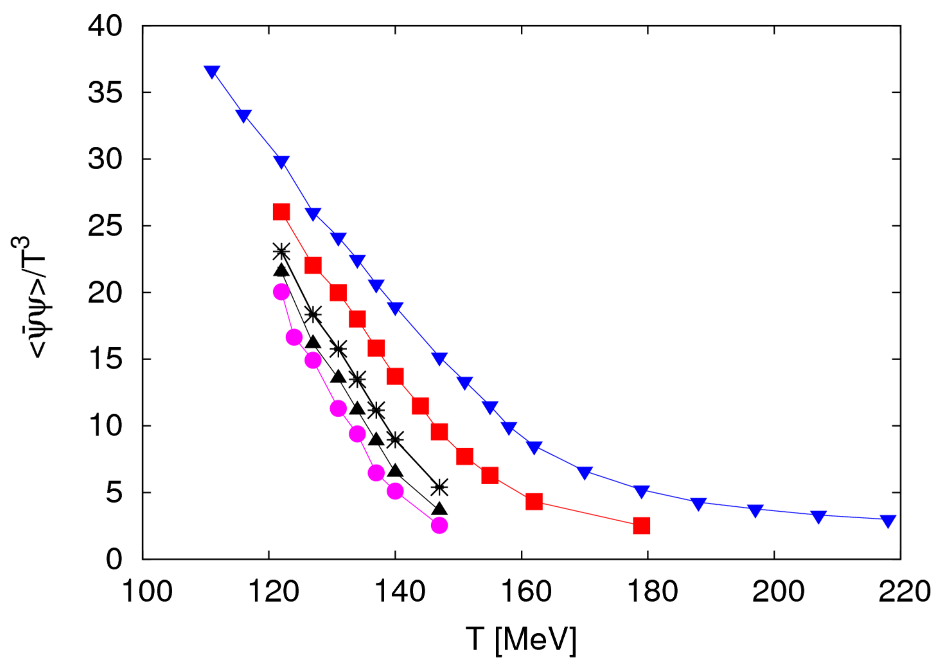
Is physics at the physical quark mass point sensitive to (universal) properties of the chiral phase transition?

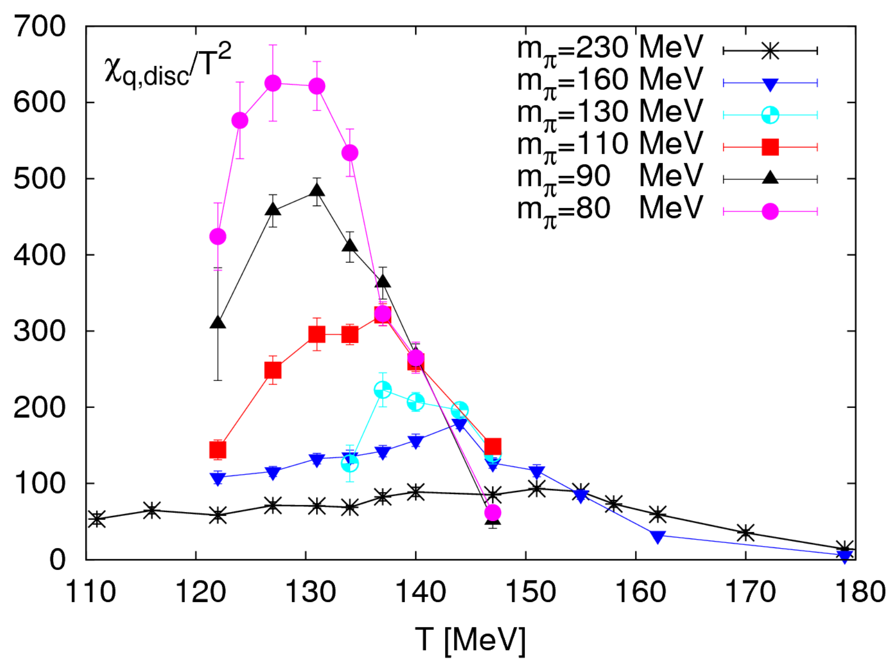
physical point may be above $\mathrm{m_s^{crit}}$

$$n_f = 3 : \ \mathrm{m}_\pi^{\mathrm{crit}} \lesssim 70\ \mathrm{MeV}$$

Nt=4, 6: improved actions
FK et al., NP(Proc.Suppl) 129 (2004) 614
G. Endrodi et al, PoS LAT 2007 (2007) 182
(also Nt=6 standard staggered)

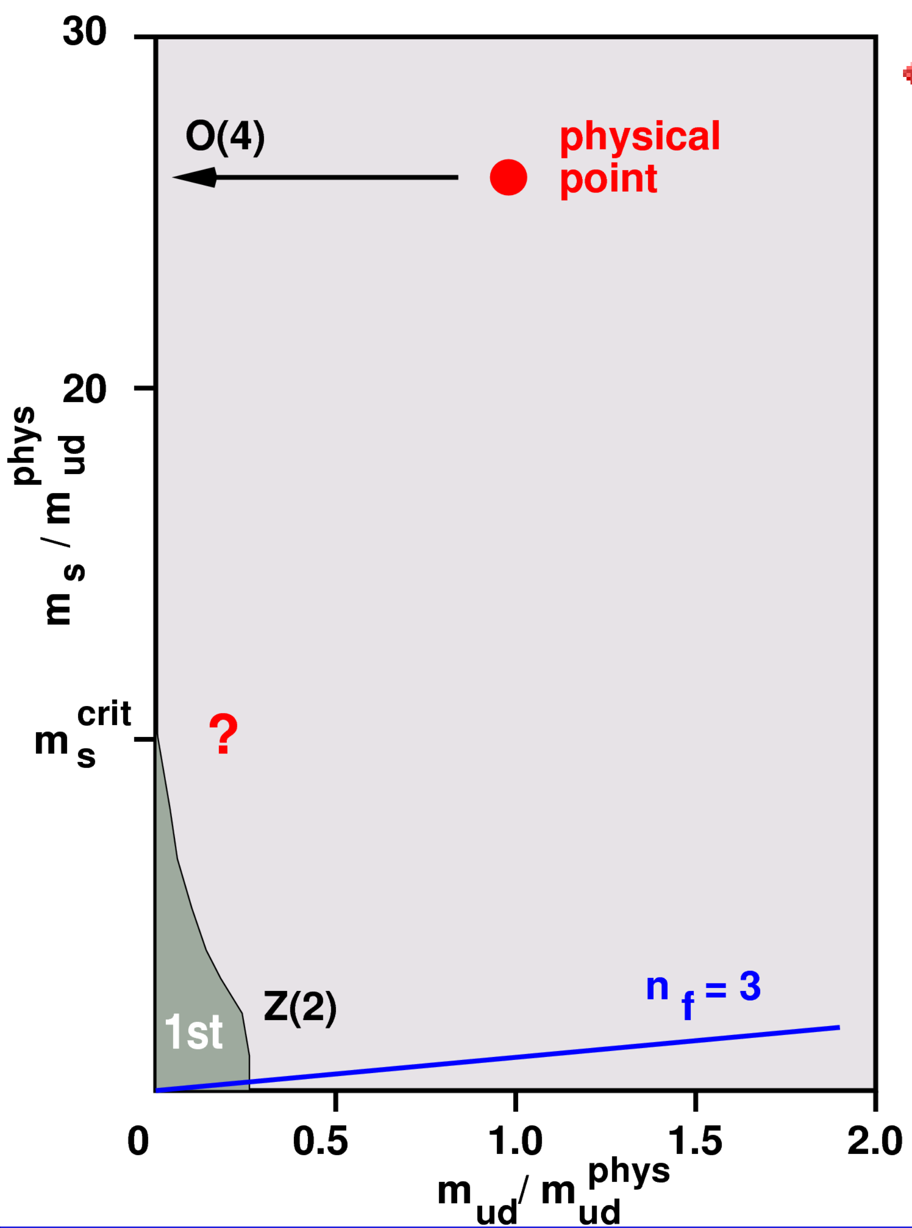
Phase transition in 3-flavor QCD





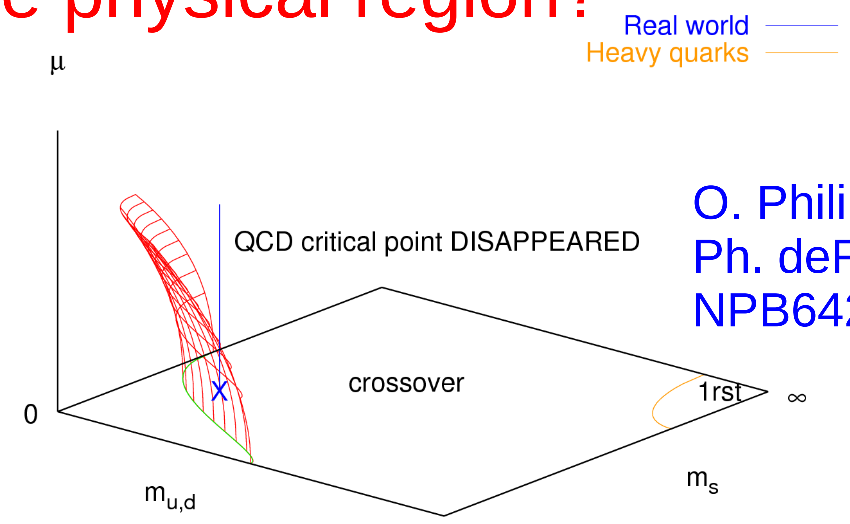
H.-T. Ding et al, in preparation

Phase diagram for $\mu_B \neq 0$



- consequence for the discussion on the existence or non-existence of a critical point:

The bending of the surface on the Z(2) boundary at the nf=3 line seems to be of little importance for the physical region?



O. Philipsen,
Ph. deForcrand,
NPB642(2002)290

Symmetries and the chiral phase transition

$$U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f)$$

$$\bar\psi \mathcal{M} \psi \sim \bar\psi_L \not{D}_\mu \psi_L + \bar\psi_R \not{D}_\mu \psi_R - m_q(\bar\psi_L \psi_R + \bar\psi_R \psi_L)$$

$U_V(1)$: baryon number $\quad \psi^\Theta = \mathrm{e}^{i\Theta}\psi \; , \; \bar\psi^\Theta = \bar\psi \mathrm{e}^{-i\Theta}$

$U_A(1)$: axial symmetry $\quad \psi^\Theta = \mathrm{e}^{i\Theta\gamma_5}\psi \; , \; \bar\psi^\Theta = \bar\psi \mathrm{e}^{i\Theta\gamma_5}$

$SU_{L,R}(n_f)$: flavor symmetry $[G_{L/R} \in U(n_f)]$

$$\psi'^i_{L/R} = G^{ij}_{L/R}\psi^j_{L/R}$$

$$\bar\psi'^j_{L/R} = \bar\psi^i_{L/R} G^{\dagger,ij}_{L/R}$$

$$\psi \equiv (\psi^u, \psi^d, ..)$$

Symmetries and the chiral phase transition

$$U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f)$$

$$\bar{\psi}\mathcal{M}\psi \sim \bar{\psi}_L \not{D}_\mu \psi_L + \bar{\psi}_R \not{D}_\mu \psi_R - m_q(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L)$$

symmetry breaking pattern: 3-d effective theory for the order parameter

R. Pisarski, F. Wilczek, PRD29 (1984) 338

$O(4)$ or $O(4) \times O(2)$

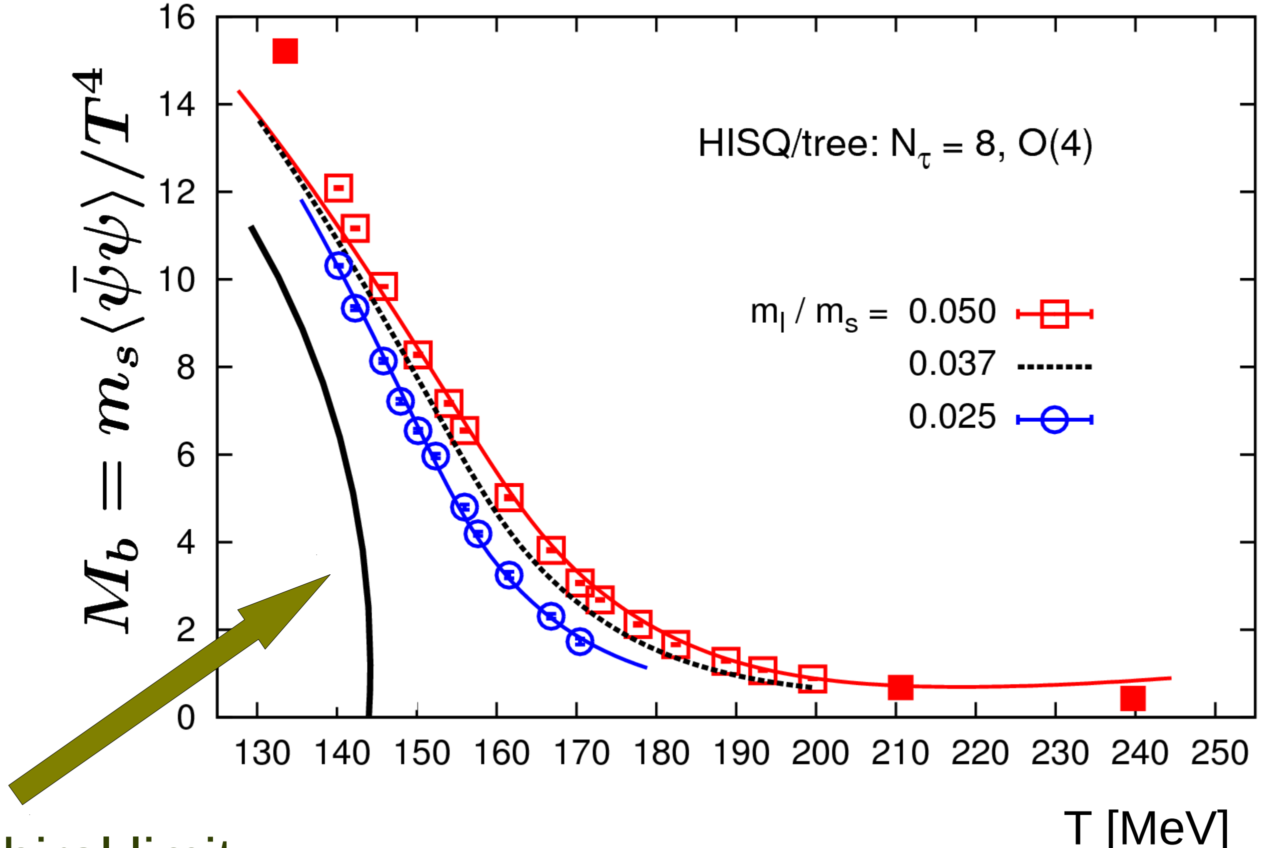
$SU(2)$

$U_A(1)$

If UA(1) is "effectively" restored at Tc, it may trigger a first order phase transition

Order parameter: chiral condensate

Is "nature" sensitive to physics in the chiral limit?



$m_\pi [MeV]$

160

140 “nature”

130

cartoon: chiral limit

A. Bazavov et al. (hotQCD), PRD85 (2012) 054503

O(N) scaling and chiral transition

- close to the chiral limit thermodynamics in the vicinity of the QCD transition is controlled by O(4) scaling functions:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{1+1/\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

- critical behavior controlled by two relevant fields: t, h
  - all couplings that do not explicitly break chiral symmetry contribute in leading order only to 't'

K. G. Wilson, Nobel prize, 1982

$$t = \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) - \kappa_q \left(\frac{\mu_q}{T}\right)^2\right) \qquad h = \frac{1}{h_0}\frac{m_l}{m_s}$$

O(N) scaling and chiral transition

- thermodynamics in the vicinity of a critical point:

$$\frac{p}{T^4} = \frac{1}{VT^3} \ln Z(V,T,\vec{\mu}) = -h^{1+1/\delta} f_s(t/h^{1/\beta\delta}) - f_r(V,T,\vec{\mu})$$

- critical behavior controlled by two relevant fields: t, h

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$$t = \frac{1}{t_0}\left(\left(\frac{T}{T_c}-1\right) - \kappa_q \left(\frac{\mu_q}{T}\right)^2 - \kappa_s \left(\frac{\mu_s}{T}\right)^2 - \kappa_{qs}\frac{\mu_q \mu_s}{T^2}\right)$$

$$h = \frac{1}{h_0}\frac{m_l}{m_s}$$

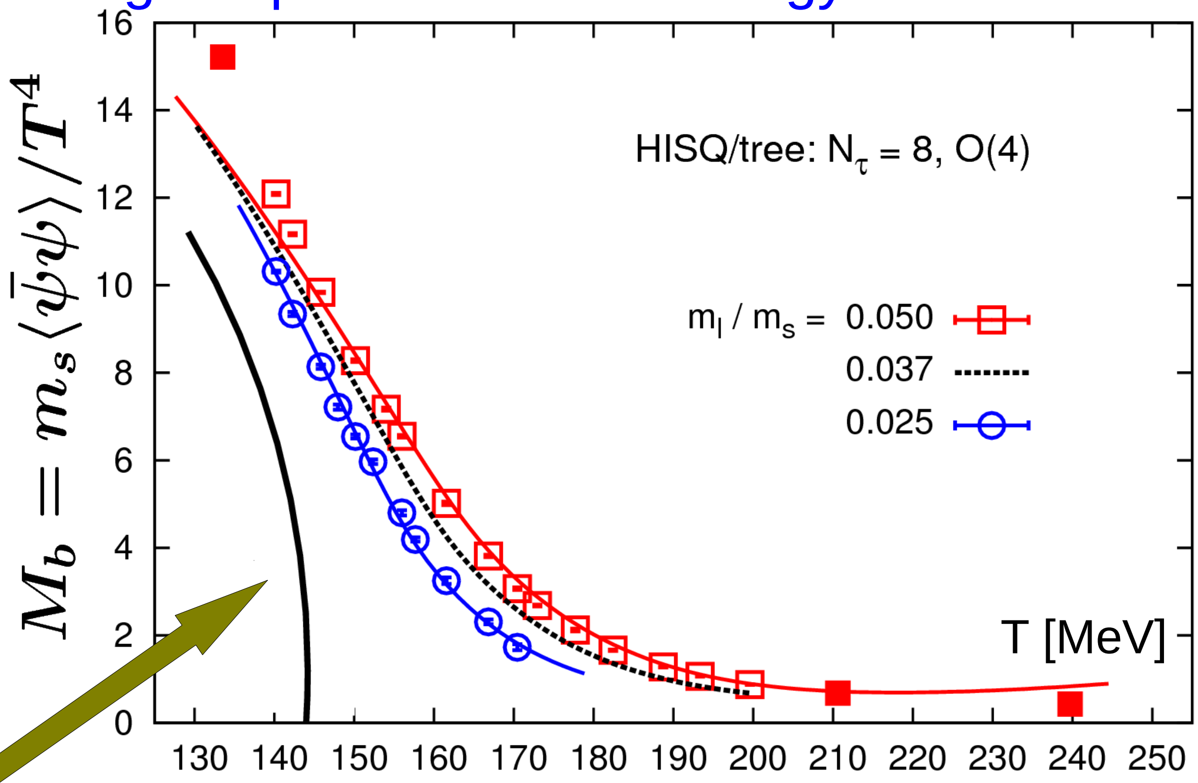
3 unique scales at vanishing chemical potential

3 additional, unique scales at non- vanishing chemical potential: curvature of the critical surface

Order parameter: chiral condensate

Is "nature" sensitive to physics in the chiral limit?

⟺ Is the order parameter controlled by the universal, singular part of the free energy ?



$m_\pi [MeV]$

160

140 “nature”

130

cartoon: chiral limit

$$M_b = \frac{\partial p/T^4}{\partial m_l/m_s} = \frac{1}{h_0} h^{1/\delta} f_G(z) + \text{regular}, \quad z = t/h^{1/\beta\delta}$$

$$f_G(z) = -(1+\frac{1}{\delta}) f_s(z) + \frac{z}{\beta\delta} f_s'(z)$$

Critical exponents

order parameter, h=0: $\langle\bar{\psi}\psi\rangle \sim (T_c - T)^\beta$

t=0: $\langle\bar{\psi}\psi\rangle \sim m_l^{1/\delta}$

specific heat, h=0: $C_V \sim |t|^{-\alpha}$

correlation length, h=0: $\xi \sim |t|^{-\gamma}$

3-d, O(4) critical exponents: $\beta = 0.380\ ,\quad \delta = 4.824\ ,$

$$\alpha = -0.2131 \longleftarrow \alpha < 0$$

scaling relations:

$$2 - \alpha = \beta(1+\delta)$$

$$\gamma = \beta(\delta - 1)$$

specific heat does not diverge in O(N) symmetric theories

Scaling properties of higher order cumulants and bulk thermodynamics

fluctuations

- Taylor expansion of the pressure

$$\frac{p}{T^4} = \sum_{n=0}^{\infty} \frac{1}{(2n)!} \chi_{B,0}^{(2n)}(T) \left(\frac{\mu_B}{T} \right)^{2n}$$

- quark number susceptibilities

$$\chi_{2n}^B = \frac{1}{VT^3} \frac{\partial^{2n} \ln Z}{\partial(\mu_B/T)^{2n}} \bigg|_{\mu_B=0}$$

bulk thermodynamics

- free energy, pressure

$$\frac{p}{T^4} = -\frac{f}{T^4} = \frac{1}{VT^3} \ln Z$$

- energy density, specific heat etc.

$$\frac{\epsilon}{T^4} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial T}$$

$$\sim -h^{(2-\alpha-n)} f_s^{(n)}(z)$$

$$+ \frac{\partial^{2n} f_r(T,V,\vec{\mu})}{\partial(\mu_B/T)^{2n}} \bigg|_{\vec{\mu}=0}$$

$$+ \frac{\partial^{n} f_r(T,V,\vec{\mu})}{\partial T^{n}} \bigg|_{\vec{\mu}=0}$$

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$$\frac{\epsilon}{T^4} = \frac{1}{VT^3} \frac{\partial \ln Z}{\partial T}$$

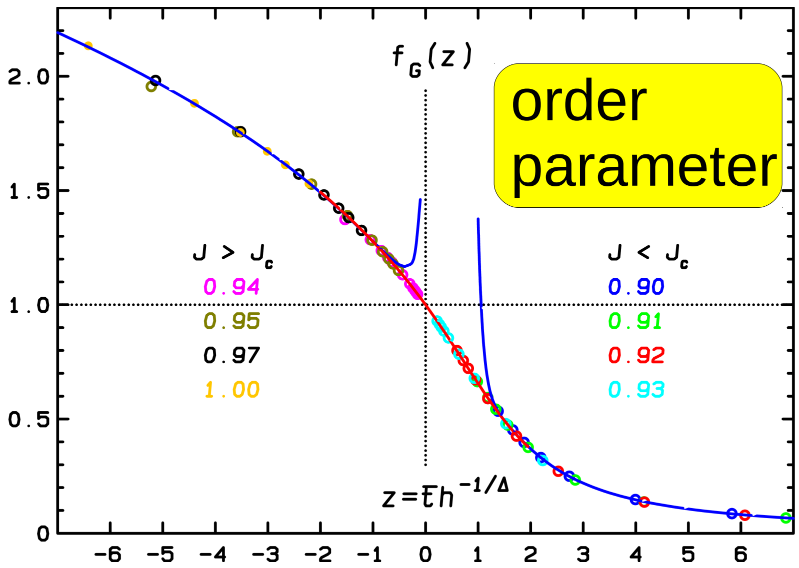
$$\sim -h^{(2-\alpha-n)} f_s^{(n)}(z)$$

diverges at Tc for m=0
only for n≥3

3d, O(4) scaling function; derivatives
of free energy scaling function

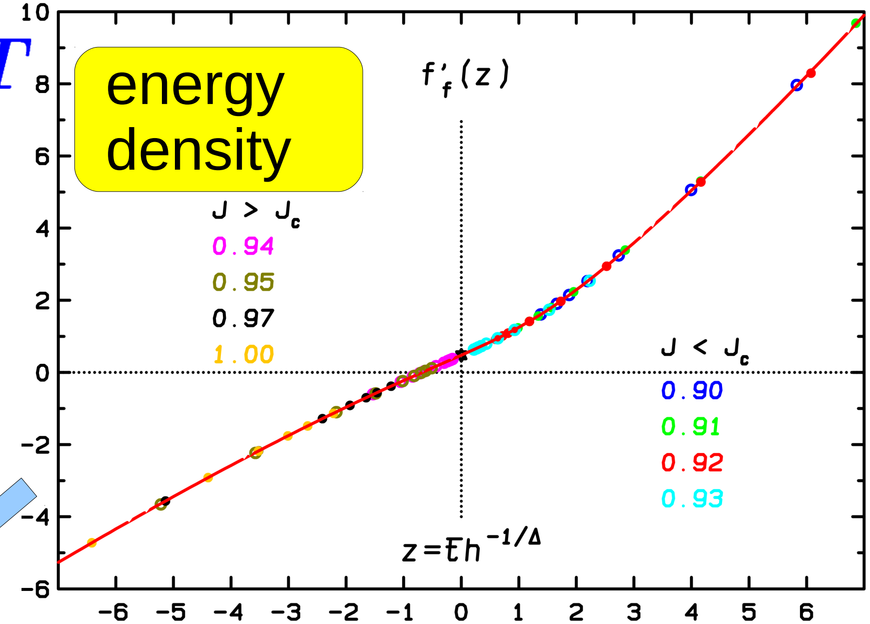
3-d, O(4) scaling functions

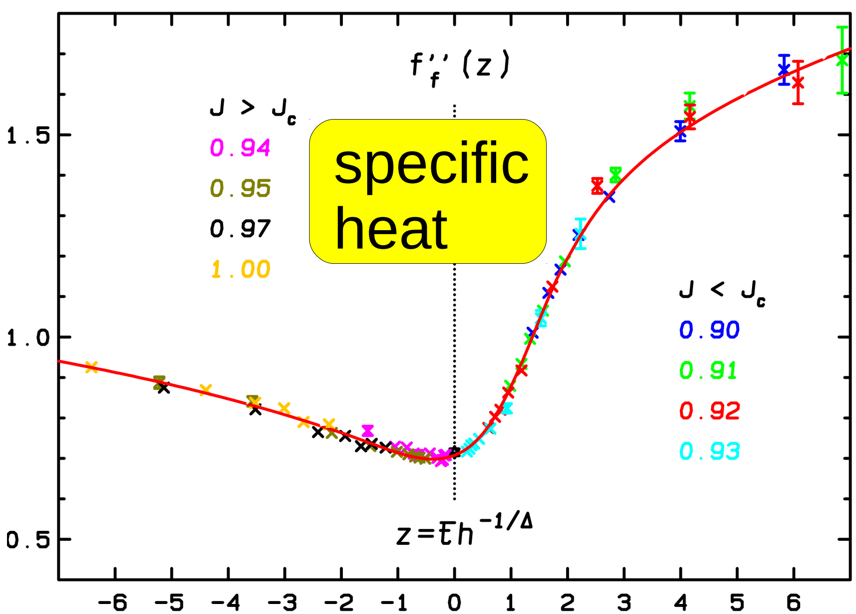
J. Engels, FK, arXiv:1105.0584

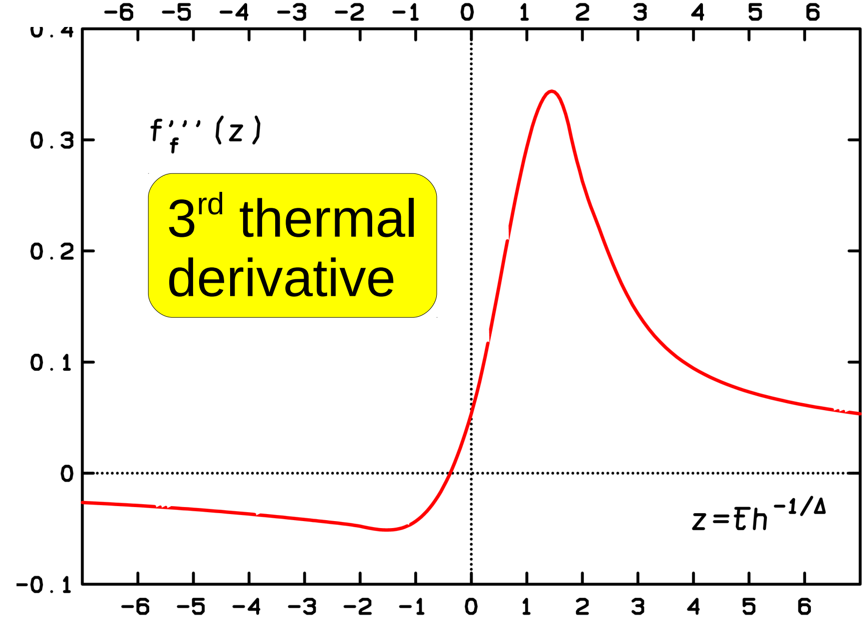


$J \equiv 1/T$

$\Delta \equiv \beta\delta$







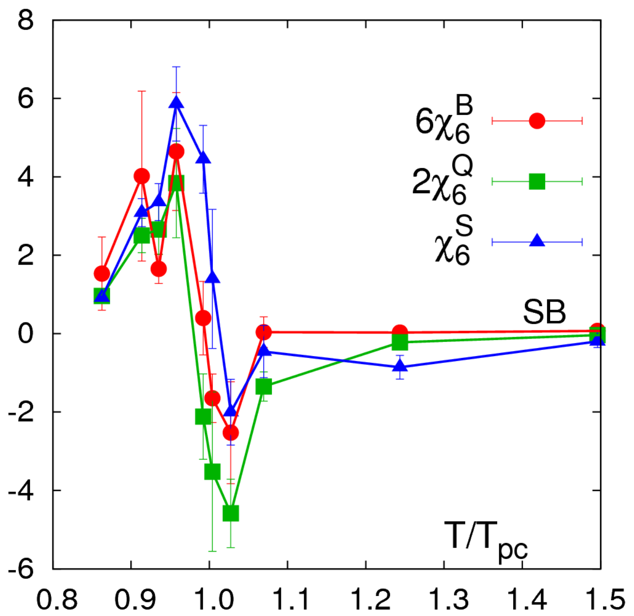
Higher order cumulants of charge fluctuation

- higher moments (e.g. 6th order) are drastically different in QCD close to criticality and in a hadron resonance gas, e.g.

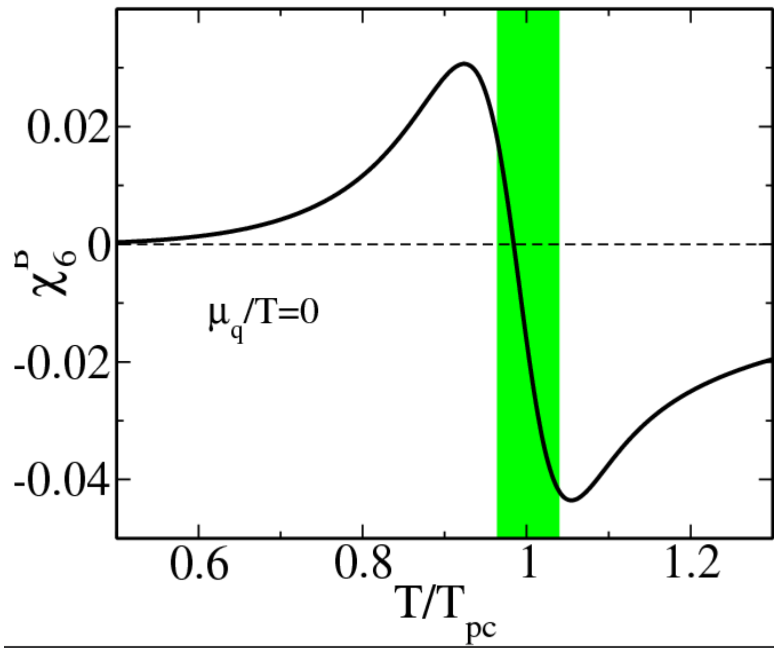
$\mu_B = 0$

$$\frac{\chi_{B,0}^{(6)}}{\chi_{B,0}^{(2)}} = \begin{cases} = 1 & \text{, hadron resonance gas} \\ < 0 & \text{, QCD at the crossover transition} \end{cases}$$

LGT: $16^3 \times 4$ (p4)



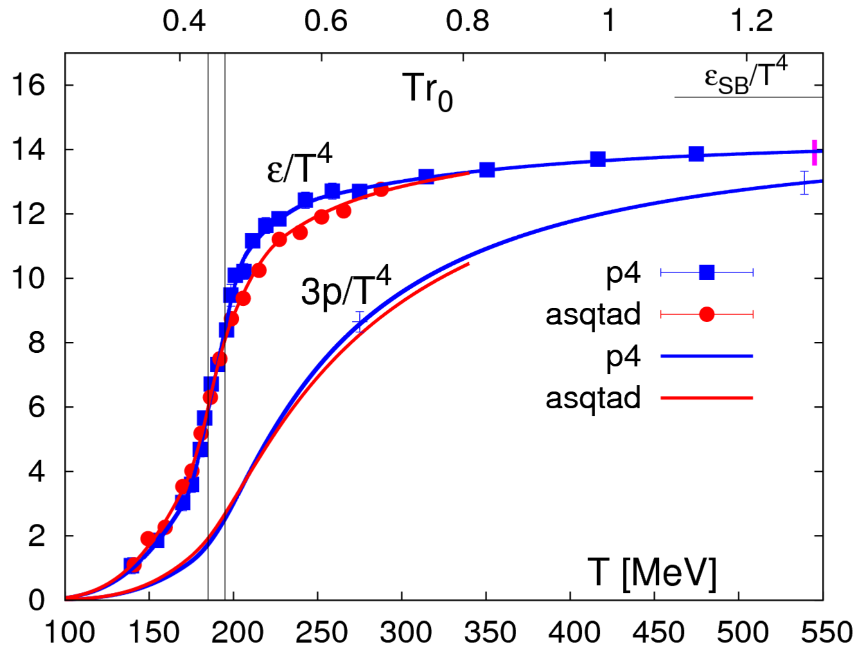
PQM model

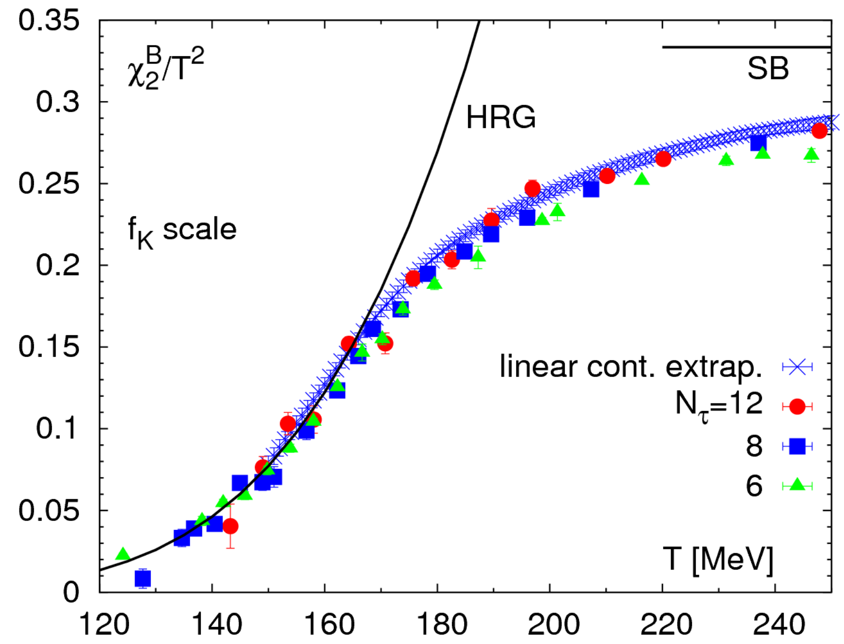


PQM model and LGT calculations reproduce expected O(4) scaling structure

B. Friman et al, arXiv:1103.3511

Energy density vs. quark number susceptibility





$$\frac{\epsilon}{T^4} = \frac{1}{VT^3}\frac{\partial \ln Z}{\partial T}$$

$$\sim -h^{(1-\alpha)} f_s'(z)$$

$$+ \left.\frac{\partial f_r(T,V,\vec{\mu})}{\partial T}\right|_{\vec{\mu}=0}$$

$$\chi_2^B = \frac{1}{VT^3}\left.\frac{\partial^2 \ln Z}{\partial(\mu_B/T)^2}\right|_{\mu_B=0}$$

$$\sim -h^{(1-\alpha)} f_s'(z)$$

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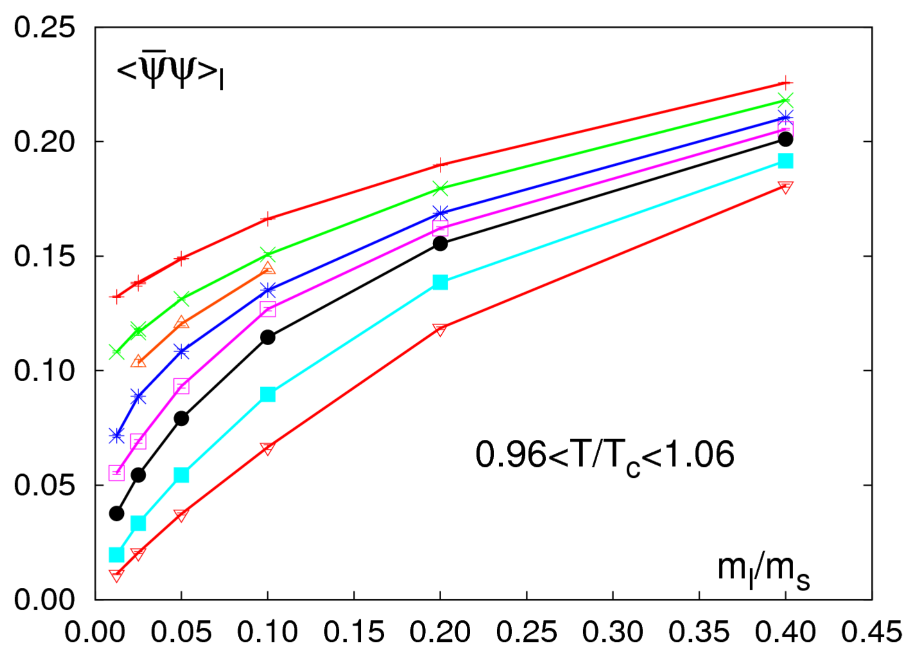
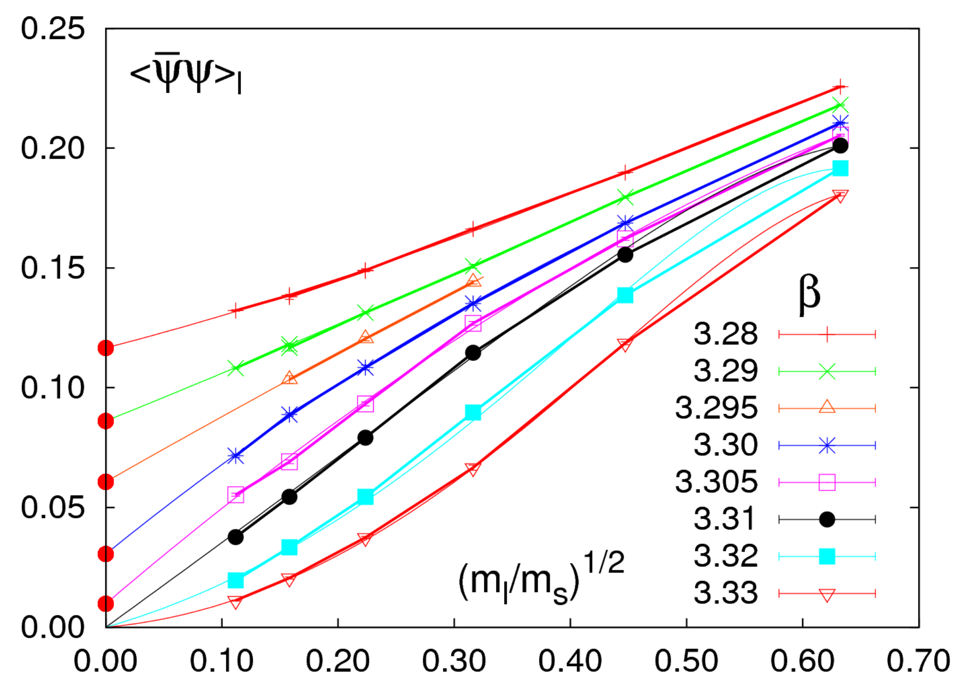
Chiral condensate
(2+1)-flavor QCD

- Goldstone modes dominate quark mass dependence of the chiral order parameter for $T \lesssim T_c \;\; \Rightarrow \;\; \langle \bar\psi\psi \rangle_l \sim m_l^{1/2}$
- analog of chiral logs at T=0





p4-action: $N_\sigma^3 \times 4 \;\; , \; N_\sigma = 8 \; - \; 32$

S. Ejiri et al (BNL-Bielefeld), Phys. Rev. D80, 094505 (2009)

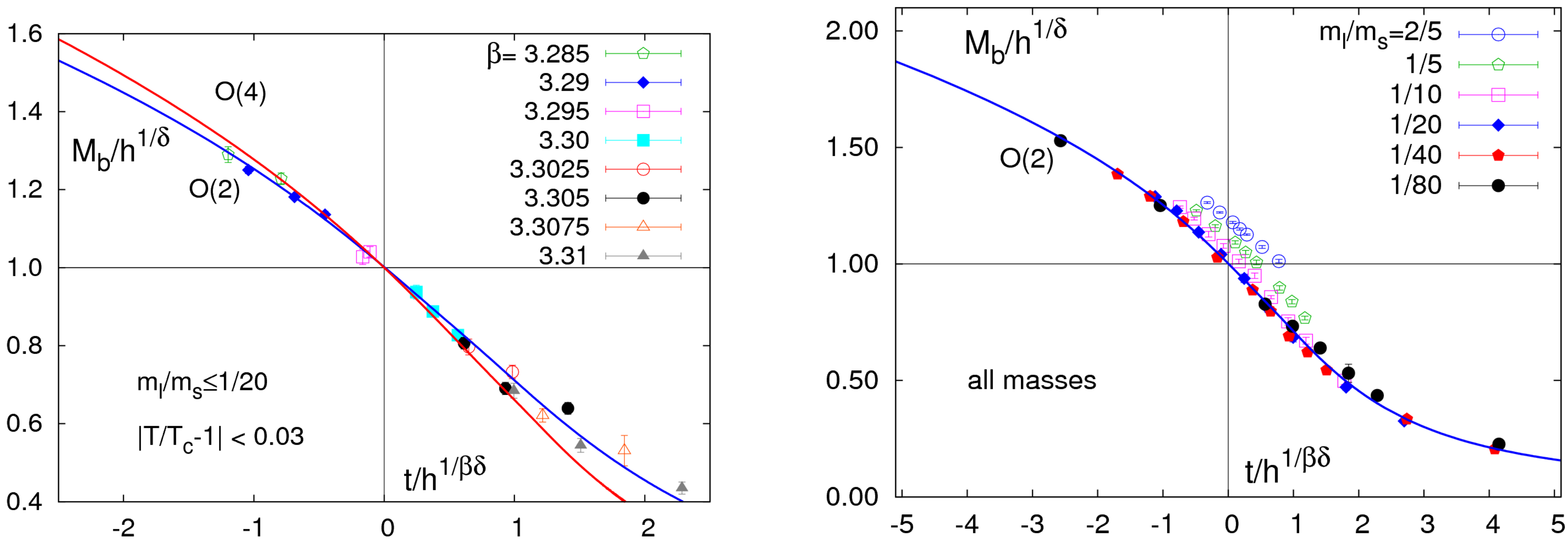
Magnetic Equation of State
(2+1)-flavor QCD

scaling of the chiral order parameter:

$$M_b \equiv \frac{m_s \langle \bar{\psi}\psi \rangle_l}{T^4} \quad , \quad z \equiv t/h^{1/\beta\delta}$$

O(2) vs. O(4)

z → 1.2z



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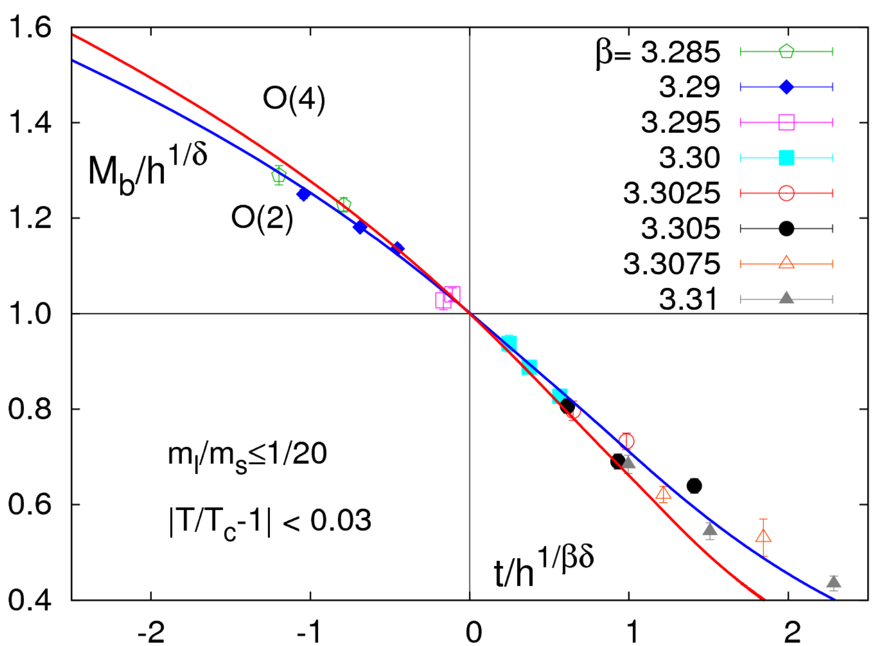
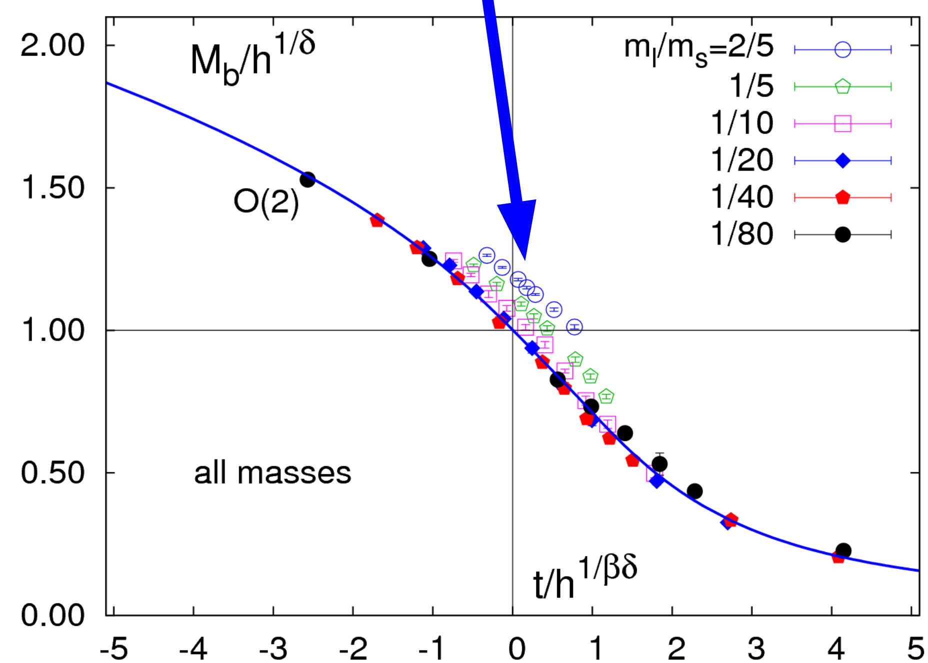
scaling violations = regular contributions are significant for $m_l/m_s \gtrsim 1/20$





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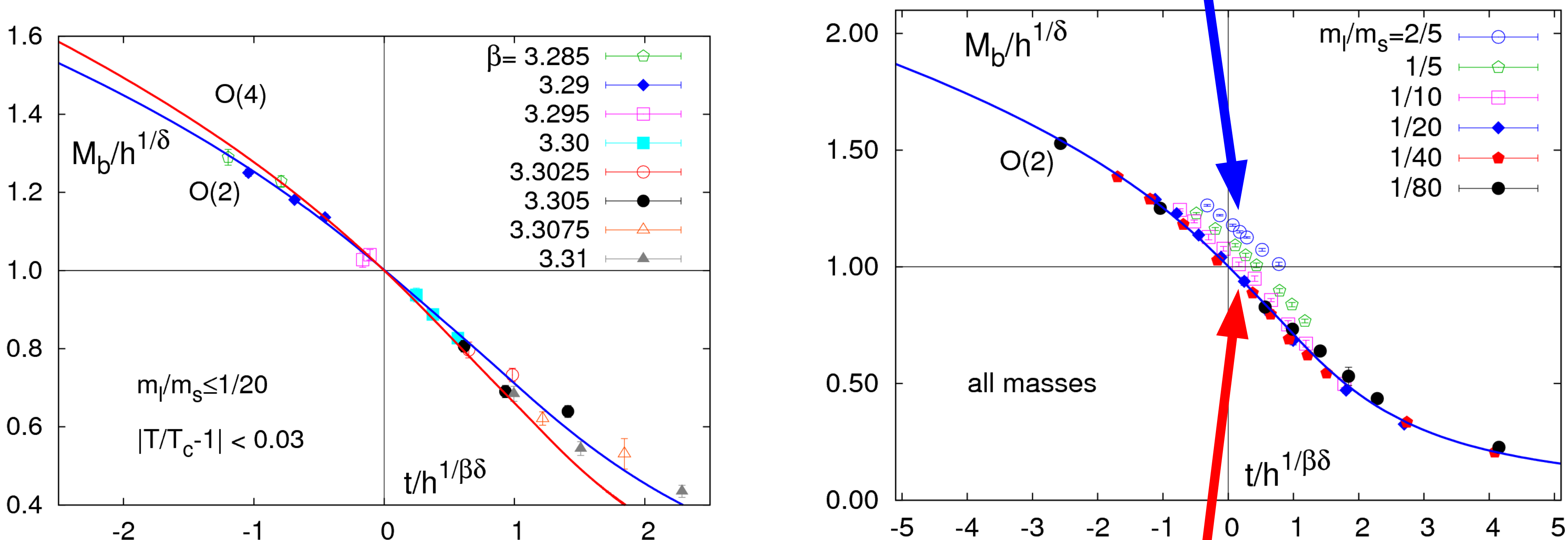

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S. Ejiri et al (BNL-Bielefeld), Phys. Rev. D80, 094505 (2009)

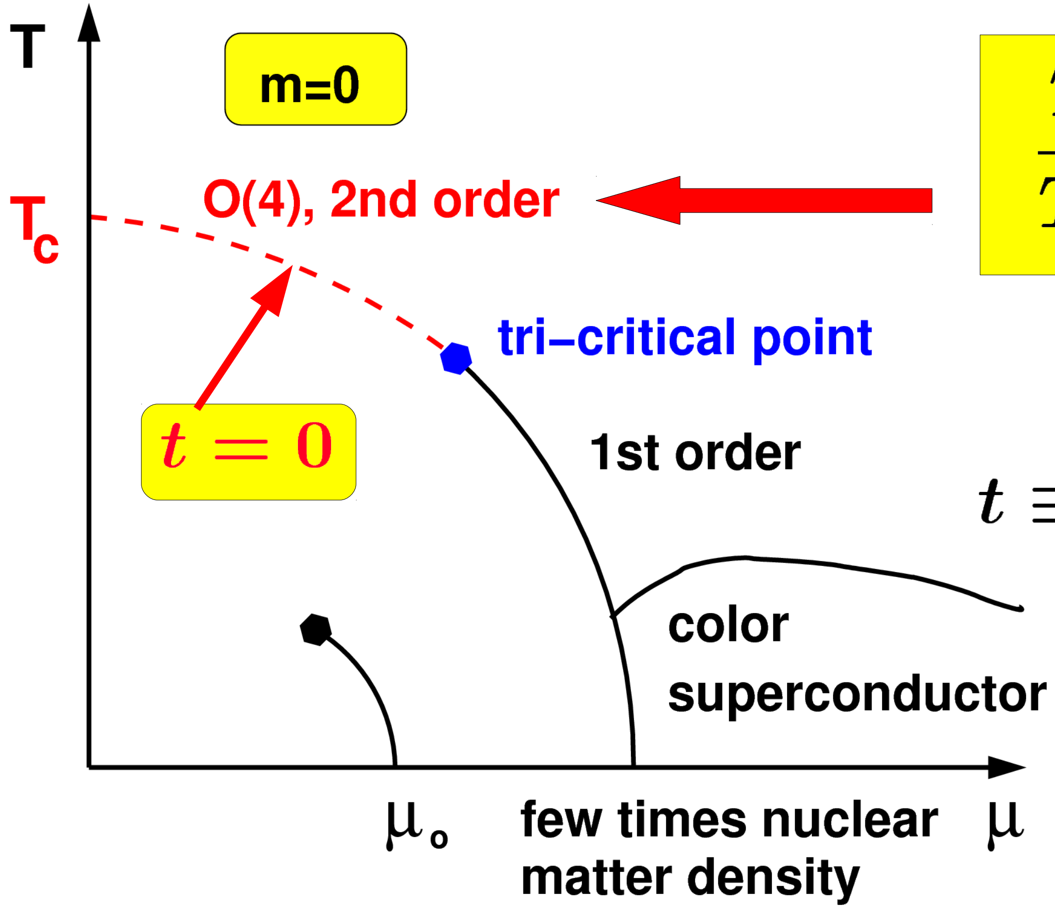
physical quark masses $m_l/m_s = 1/27$ seem to be in the scaling regime

The curvature of the critical line

BNL-Bielefeld, arXiv:1011.3130

- QCD, chiral limit (u,d quarks only)
- use scaling relations to extract the curvature of $T_c(\mu_B)$

$$\mu_u = \mu_d > 0,\ \mu_Q = \mu_s = 0,\ \ \mu_B = 3\mu_q$$



$$\boxed{\frac{T}{T_c} = 1 - \kappa_q \left(\frac{\mu_q}{T}\right)^2}$$

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c} - 1\right) - \kappa_q \left(\frac{\mu_q}{T}\right)^2\right)$$

scaling laws control curvature of chiral transition line for small μ_q/T

The curvature of the critical line

BNL-Bielefeld, arXiv:1011.3130

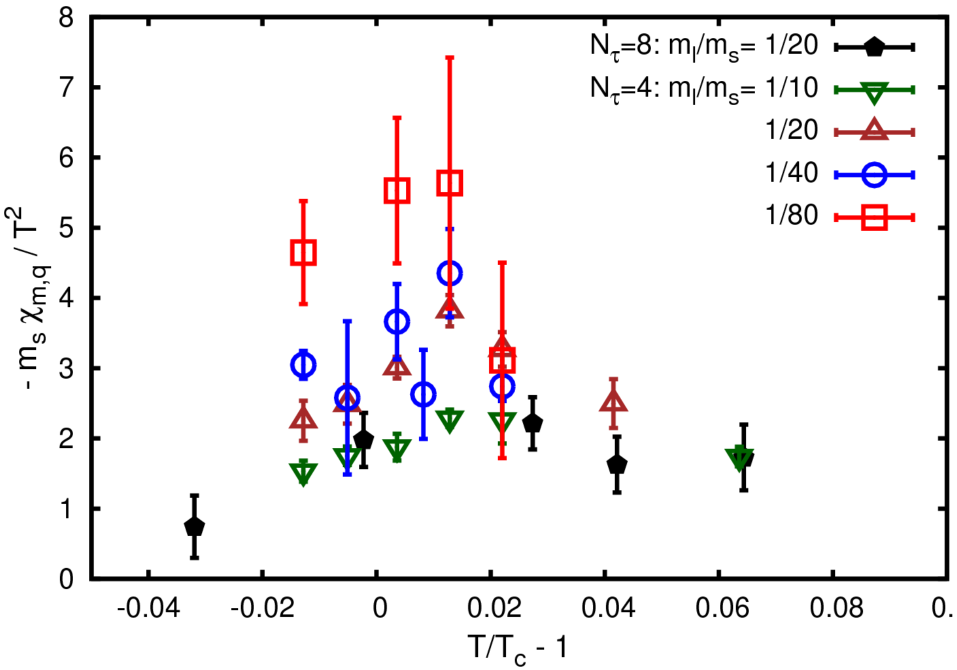
- "thermal" fluctuations of the order parameter

$$t \equiv \frac{1}{t_0}\left(\left(\frac{T}{T_c}-1\right)-\kappa_q\left(\frac{\mu_q}{T}\right)^2\right)\ ,\ z=t/h^{1/\beta\delta}$$

$$M_b \equiv \frac{m_s\langle\bar\psi\psi\rangle}{T^4} = h^{1/\delta} f_G(z) \quad \text{fixes}\ \ T_c,\ t_0,\ h_0$$

$$\frac{\chi_{m,q}}{T} = \frac{\partial^2\langle\bar\psi\psi\rangle/T^3}{\partial(\mu_q/T)^2}$$

$$= \frac{2\kappa_q T}{t_0 m_s} h^{(\beta-1)/\delta\beta} f'_G(z)$$

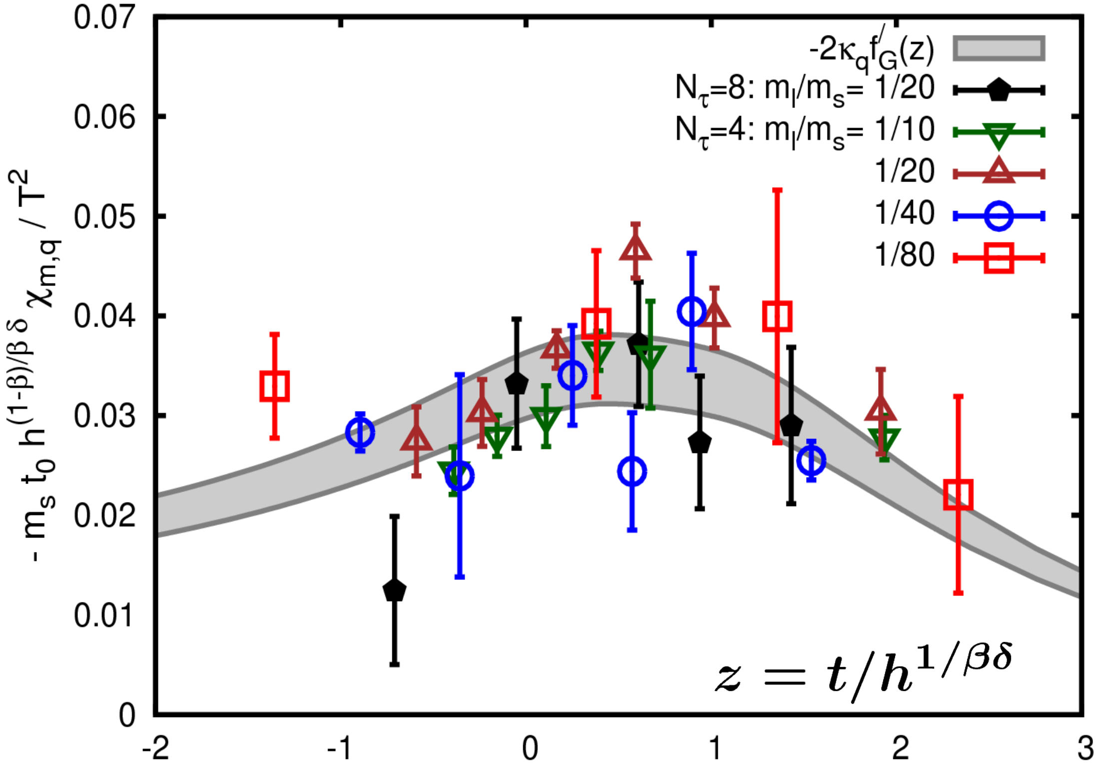


The curvature of the critical line

BNL-Bielefeld, arXiv:1011.3130

- "thermal" fluctuations of the order parameter determine universal curvature coefficient at small μ_q/T

$$\frac{t_0 m_s \chi_{m,q}}{T^2} = \partial^2 M_b/\partial(\mu_q/T)^2 = 2\kappa_q h^{(\beta-1)/\delta\beta} f'_G(z)$$

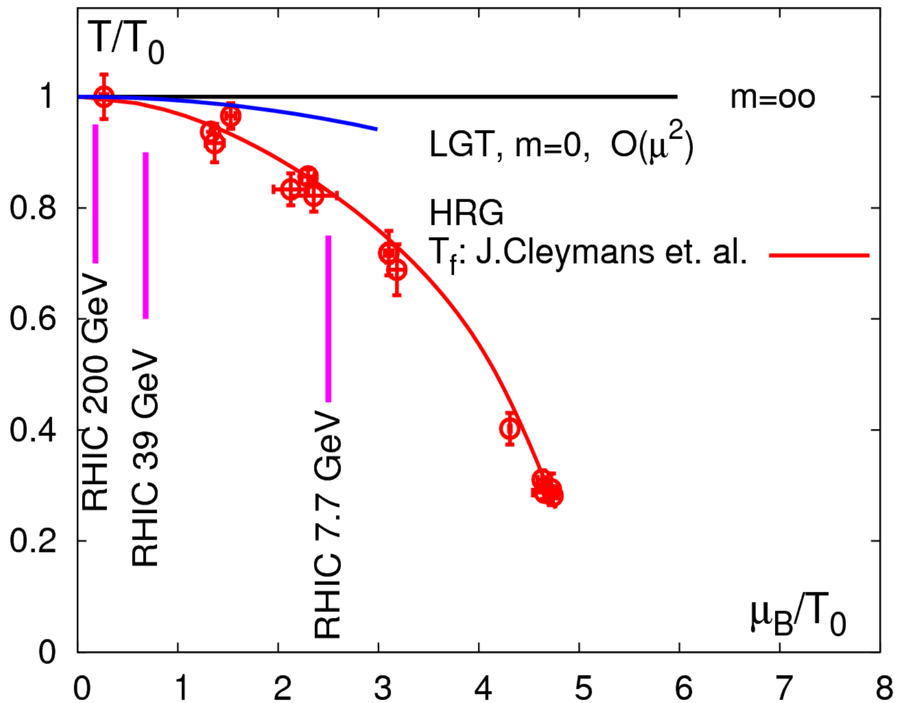


analysis for 2 values of the cut-off and 4 (1) different quark masses

$$\kappa_q = 0.059 \pm 0.006$$

compare to freeze-out curve

Chiral Transition and Freeze-out



chiral phase transition curve: t=0

$$\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7)\left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

freeze-out curve in heavy ion collisions:

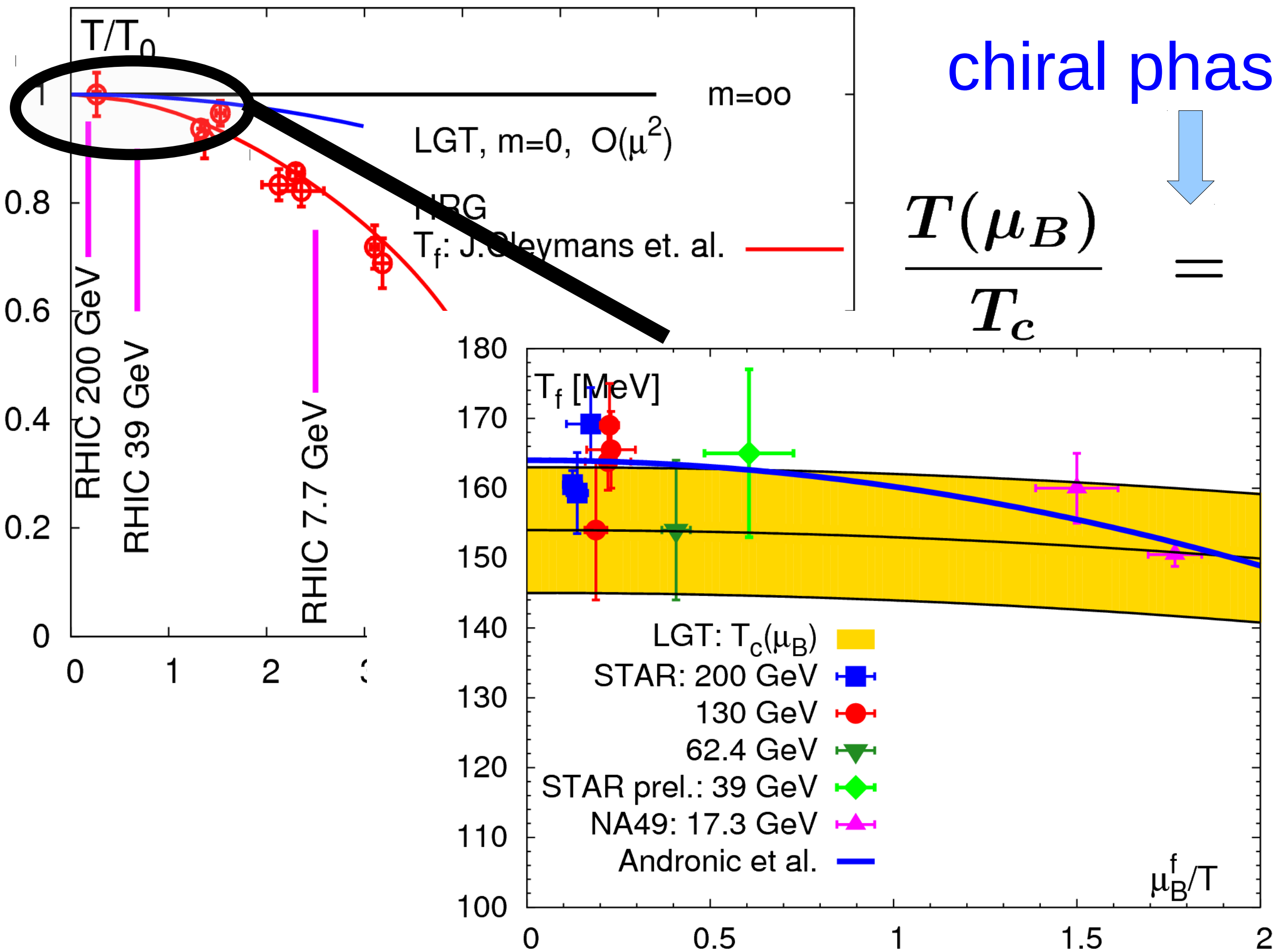
$$\frac{T(\mu_B)}{T_c} = 1 - 0.023\left(\frac{\mu_B}{T}\right)^2 - c\left(\frac{\mu_B}{T}\right)^4$$

$$\mu_B(\sqrt{s_{NN}}) = \frac{d}{1 + e\sqrt{s_{NN}}}$$

open issues:
- continuum limit
- strangeness conservation
- non zero charge

J. Cleymans et al., Phys.Rev. C73, 034905 (2006)

Chiral Transition and Freeze-out



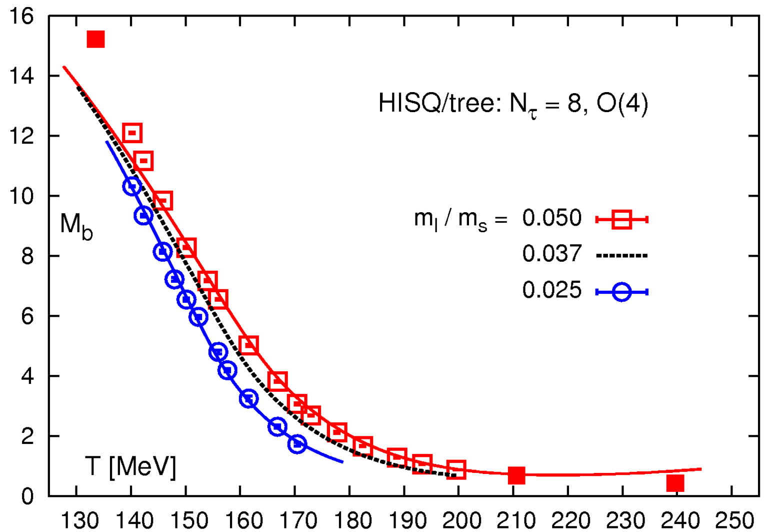
chiral phase transition curve: t=0

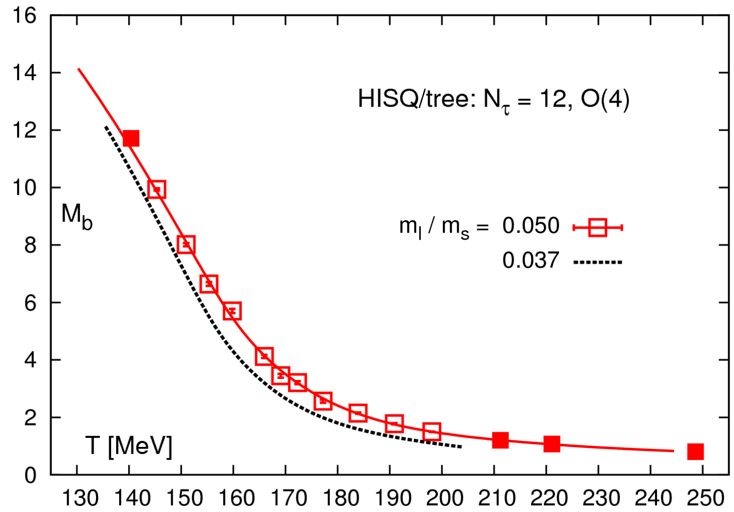
$$\frac{T(\mu_B)}{T_c} = 1 - 0.0066(7) \left(\frac{\mu_B}{T}\right)^2 + \mathcal{O}(\mu_B^4)$$

phenomenological freeze-out curve, QCD critical line and experimental data (obtained by assuming the validity of the HRG model) are consistent for

$$\mu_B/T \lesssim 2$$

The Chiral Transition Temperature





– use three different lattice sizes (lattice spacings) to perform a continuum extrapolation

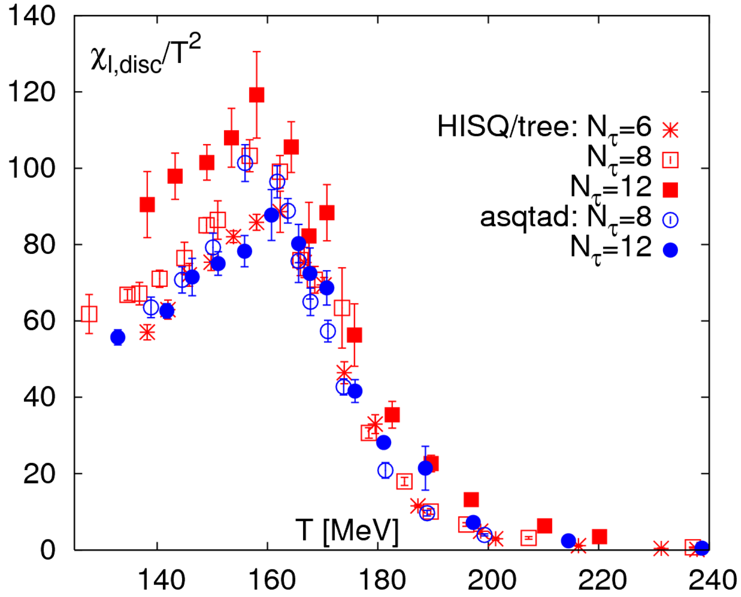
– use scaling relations to interpolate/ extrapolate to physical quark masses

– locate pseudo-critical temperature from quark number susceptibility

$$\chi_{m,l}(T) = \frac{\partial\langle\bar\psi\psi\rangle}{\partial m_l}$$
$$= \chi_{l,disc} + \chi_{l,con}$$

A. Bazavov et al. (hotQCD), PRD85 (2012) 054503

The Chiral Transition Temperature



– locate pseudo-critical temperature from quark number susceptibility

$$\chi_{m,l}(T) = \frac{\partial \langle \bar\psi \psi \rangle}{\partial m_l} = \chi_{l,disc} + \chi_{l,con}$$

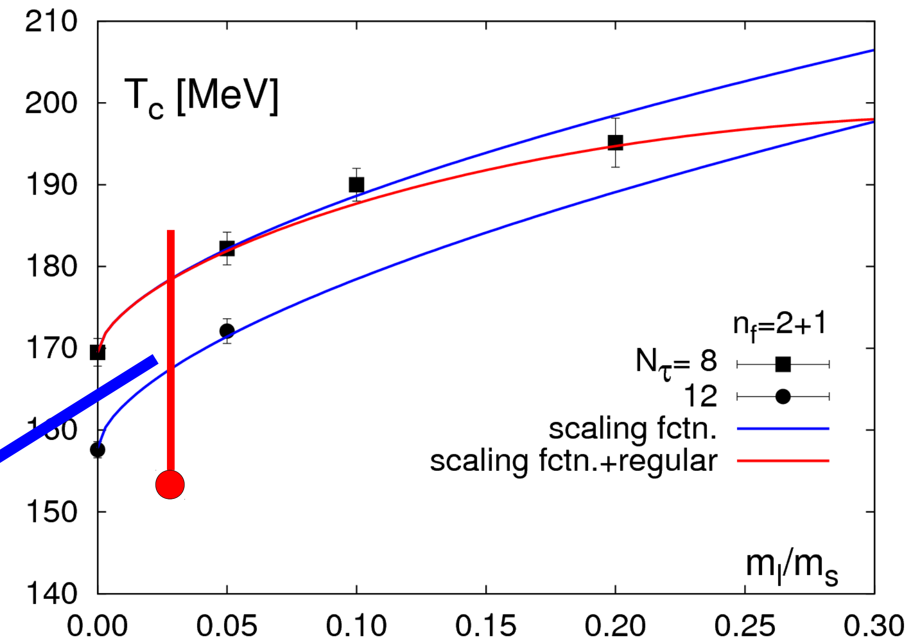
$$\frac{m_s^2 \chi_{m,l}}{T^4} = \left(\frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) + \frac{\partial f_{M,reg}(T,H)}{\partial H} \right)$$

$$\text{with} \;\; f_\chi(z) = \frac{1}{\delta}[f_G(z) - \frac{z}{\beta} f'_G(z)].$$

– quark mass dependence of peak position of $\chi_{m,l}$ is a universal scaling function

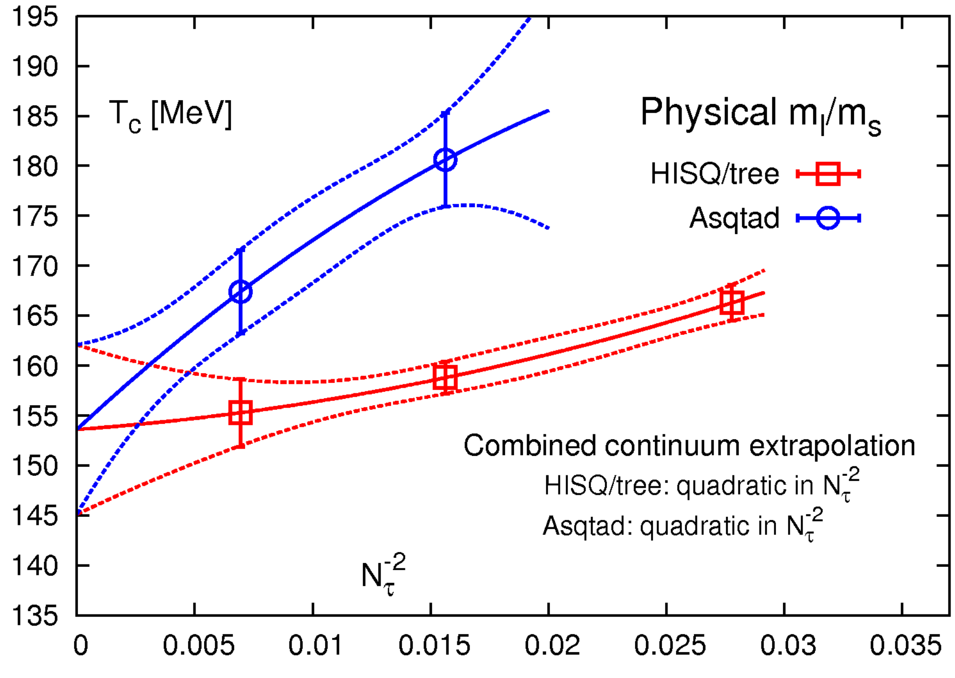
$$z = z_{max} \Rightarrow \frac{T_c(m_q) - T_c(0)}{T_c(0)} = c \cdot \left(\frac{m_l}{m_s} \right)^{1/\beta\delta} + reg.$$

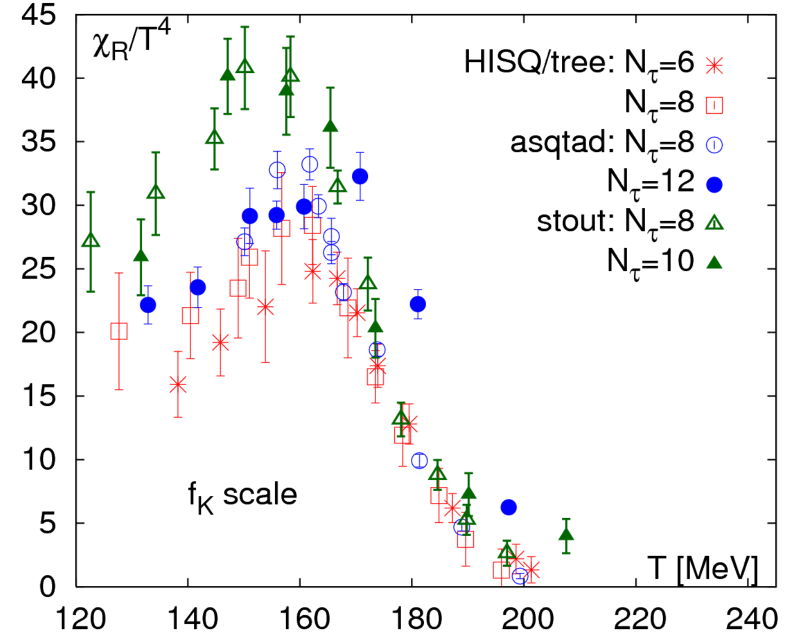
The Chiral Transition Temperature



for physical values of the quark masses scaling violations in T_c are small $\Leftrightarrow$ the crossover temperature reflects chiral dynamics

$$T_c = (154 \pm 8 \pm 1)\mathrm{MeV}$$





Symmetries and in-medium properties of hadrons

Which symmetries are restored at Tc?

- thermal hadron correlation functions

Greens functions G of quark-antiquark in different quantum number channels H, controlled by operators J

$$J_H(x) = \bar{q}(x)\Gamma_H q(x) \qquad \Gamma_H = 1,\ \gamma_5,\ \gamma_\mu,\ \gamma_\mu\gamma_5$$

scalar, pseudo-scalar, vector, axial-vector

$$q(\bar{q}) = u(\bar{u}),\ d(\bar{d}), ... \Rightarrow$$

$\bar{q}q = \bar{u}u$ flavor singlet

$\bar{q}q = \bar{u}d$ flavor non-singlet

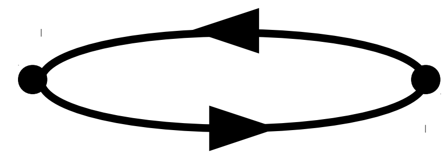
$$G_H(\tau, \vec{x}) = \langle J_H(\tau, \vec{x})\ J_H^\dagger(0, \vec{0})\rangle \ \sim\ \mathrm{e}^{-m_H^{scr}|\vec{x}|}$$

screening mass

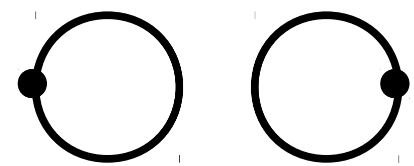
Thermal modification of the hadron spectrum

quark propagator: $\bar{q}(x)q(0) = \mathrm{tr} M_q^{-1}(x,0)$

connected



$$
\begin{aligned}
G_\pi(x) &= \mathrm{tr}\langle\, \gamma_5 M_l^{-1}(x,0)\gamma_5 M_l^{-1}(0,x)\,\rangle \\
G_\eta(x) &= G_\pi(x) - \langle \mathrm{tr}\left[\gamma_5 M_l^{-1}(x,x)\right] \mathrm{tr}\left[\gamma_5 M_l^{-1}(0,0)\right]\rangle \\
G_\delta(x) &= -\mathrm{tr}\langle\, M_l^{-1}(x,0) M_l^{-1}(0,x)\,\rangle \\
G_\sigma(x) &= G_\delta(x) + \langle \mathrm{tr} M_l^{-1}(x,x) \mathrm{tr} M_l^{-1}(0,0)\rangle \\
&\quad - \langle \mathrm{tr} M_l^{-1}(x,x)\rangle\ \langle \mathrm{tr} M_l^{-1}(0,0)\rangle
\end{aligned}
$$



disconnected

hadronic susceptibilities

$$\chi_\pi = \sum_x G_\pi(x) \equiv \chi_{5,\mathrm{con}} \quad , \quad \chi_\delta = \sum_x G_\delta(x) = \chi_{\mathrm{con}}$$

$$\chi_\eta = \sum_x G_\eta(x) \equiv \chi_{5,\mathrm{con}} - \chi_{5,\mathrm{disc}}.$$

$$\chi_\sigma = \sum_x G_\sigma(x) = \chi_{\mathrm{con}} + \chi_{\mathrm{disc}}$$

Thermal modification of the hadron spectrum

$T < T_c$ **:** broken chiral symmetry is reflected in the hadron spectrum

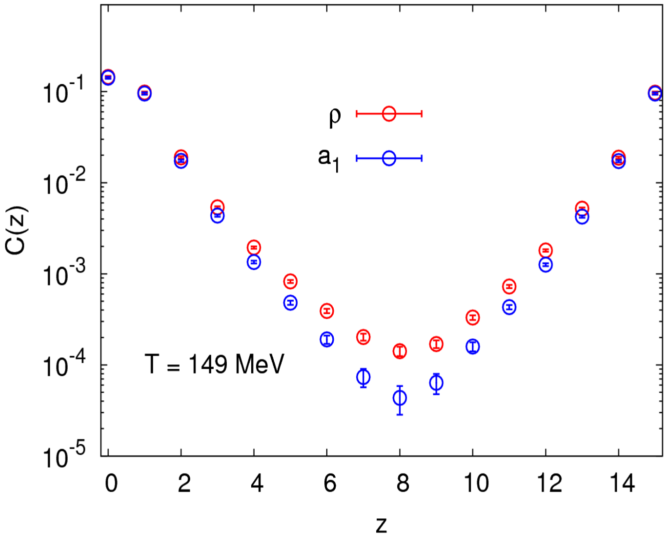
	a_1: $\bar{q}\gamma_5\gamma_\mu q$	$\xleftrightarrow{}$ $SU(2)_L \times SU(2)_R$	ρ : $\bar{q}\gamma_\mu q$	
$\chi_{5,con}$	π : $\bar{q}\gamma_5 \frac{\tau}{2} q$	$\longleftrightarrow$	σ : $\bar{q} q$ (f_0)	$\chi_{con} + \chi_{disc}$
flavor non-singlet	$U(1)_A$ $\updownarrow$		$\updownarrow$ $U(1)_A$	flavor singlet
χ_{con}	δ : $\bar{q}\frac{\tau}{2} q$ (a_0)	$\longleftrightarrow$ $SU(2)_L \times SU(2)_R$	η : $\bar{q}\gamma_5 q$	$\chi_{5,con} - \chi_{5,disc}$

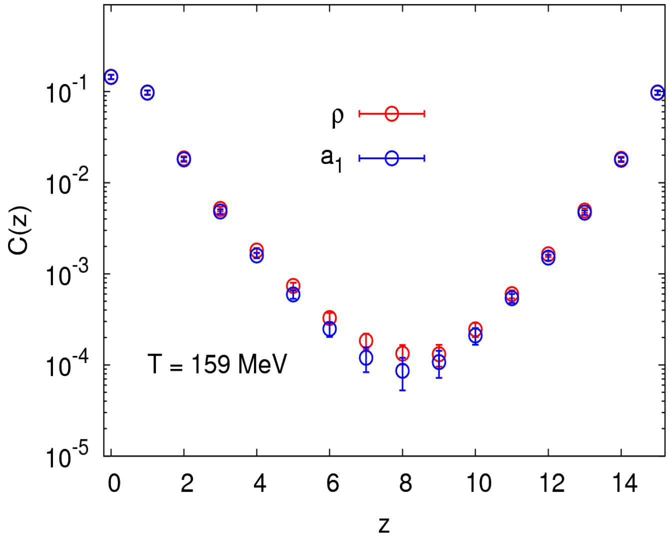
$T \geq T_c$ **:** restoration of symmetries is reflected in the (thermal) hadron spectrum

$SU(2)_L \times SU(2)_R$ **:** $(\pi,\ \sigma),\ (a_1, \rho)$ degenerate

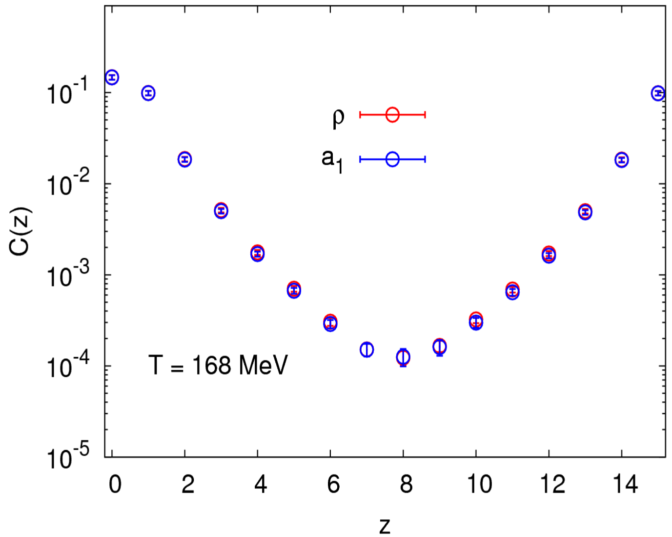
$U(1)_A$ **:** $(\pi,\ \delta)$ degenerate

Symmetry restoration and correlation functions





$$C_H(z) = \sum_{x,y,\tau} G_H(x,y,z,\tau)$$



Thermodynamics with domain wall fermions, hotQCD, arXiv:0512xxxx

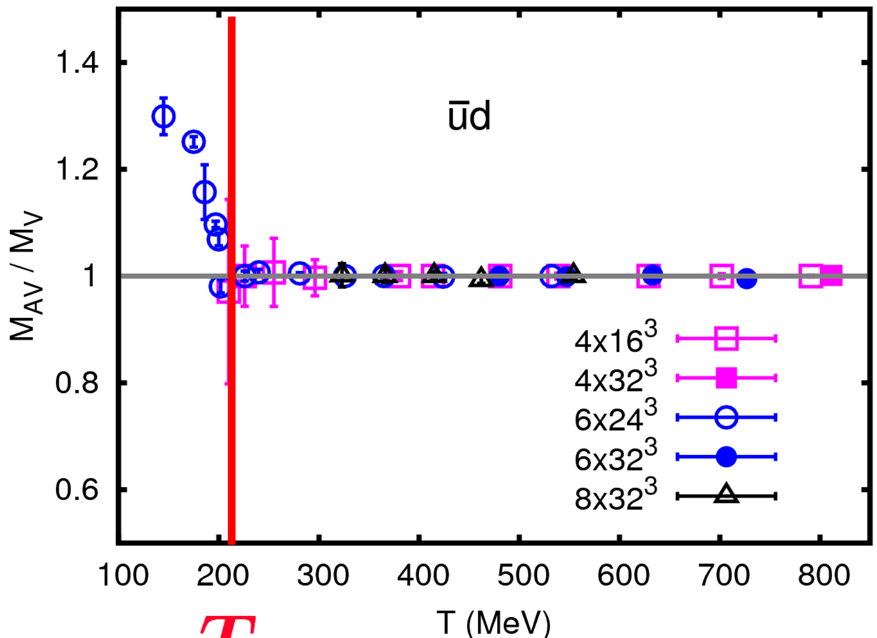
chiral flavor symmetry is restored at

$T \gtrsim 160$ MeV $\quad m_\pi \simeq 200$ MeV

no cont. extrap.

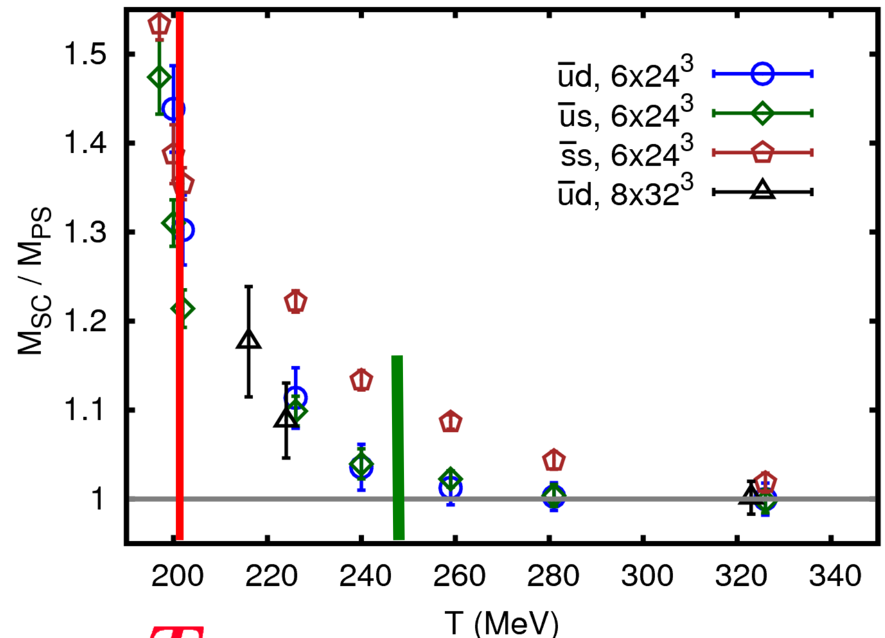
Chiral (flavor) symmetry restoration

$SU(2)_L \times SU(2)_R$:



T_c

$U(1)_A$:



T_c

$\sim 1.2T_c$

- chiral flavor symmetry restored at T_c ;
- $U_A(1)$ stays broken, but is "effectively" restored at about $1.2T_c$

caveat: (i) calculation done with $m_\pi \simeq 200\mathrm{MeV}$
(ii) staggered fermions

What about UA(1) restoration?

Restoration of the axial symmetry

$T < T_c$: broken chiral symmetry is reflected in the hadron spectrum

$$\begin{tikzcd} \chi_{5,con} & \pi : \bar{q}\gamma_5\frac{\tau}{2}q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma : \bar{q}q \; (f_0) \arrow[d, leftrightarrow, "U(1)_A"] & \chi_{con}+\chi_{disc} \\ \chi_{con} & \delta : \bar{q}\frac{\tau}{2}q \; (a_0) \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"'] & \eta : \bar{q}\gamma_5 q & \chi_{5,con}-\chi_{5,disc} \end{tikzcd}$$

$T \geq T_c$: $SU(2)_L \times SU(2)_R$ restored

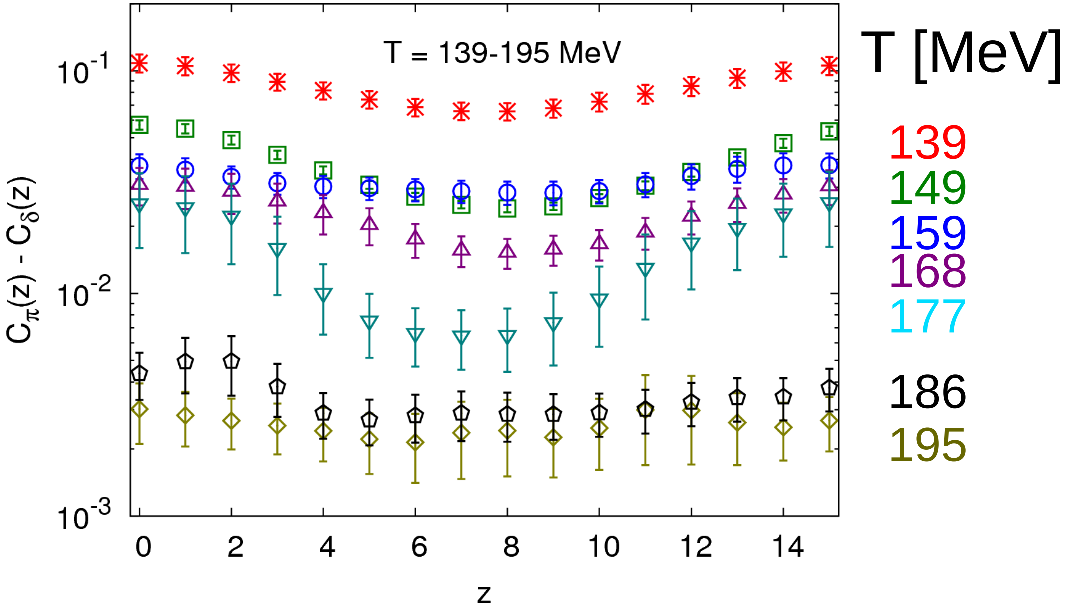
$$\Rightarrow \quad \chi_{5,con} = \chi_{con} + \chi_{disc}$$

$U(1)_A$ restoration $\Leftrightarrow \chi_{disc} = 0$

$$\Leftrightarrow G_\pi(x) - G_\sigma(x) = 0$$

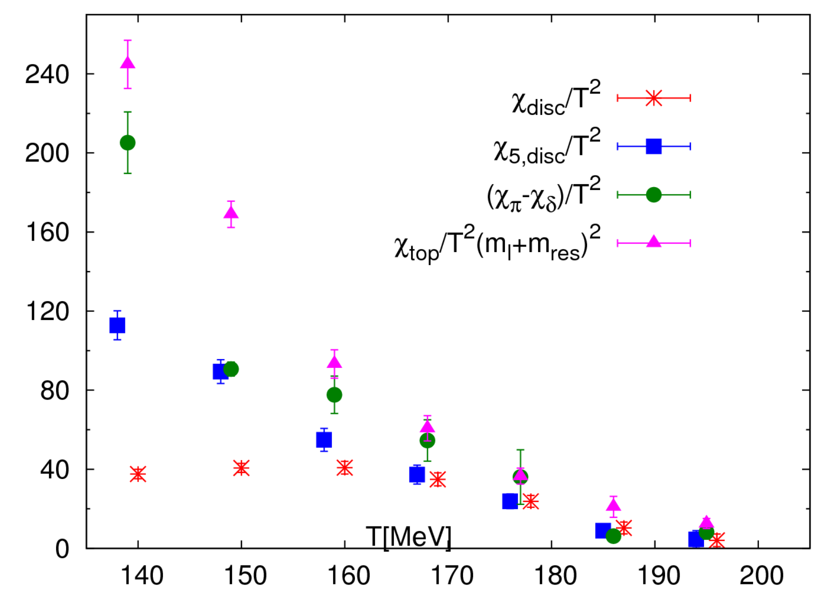
U(1)$_A$ remains broken

the difference of the scalar and pseudo-scalar drops by an order of magnitude but stays non-zero



above Tc (but still for m>0):

$$\frac{\chi_\pi - \chi_\delta}{T^2} = \frac{\chi_{disc}}{T^2} = \frac{\chi_{5,disc}}{T^2} > 0$$



thermodynamics with domain wall fermions

hotQCD, arXiv:0512.xxxx

U(1)$_A$ remains broken

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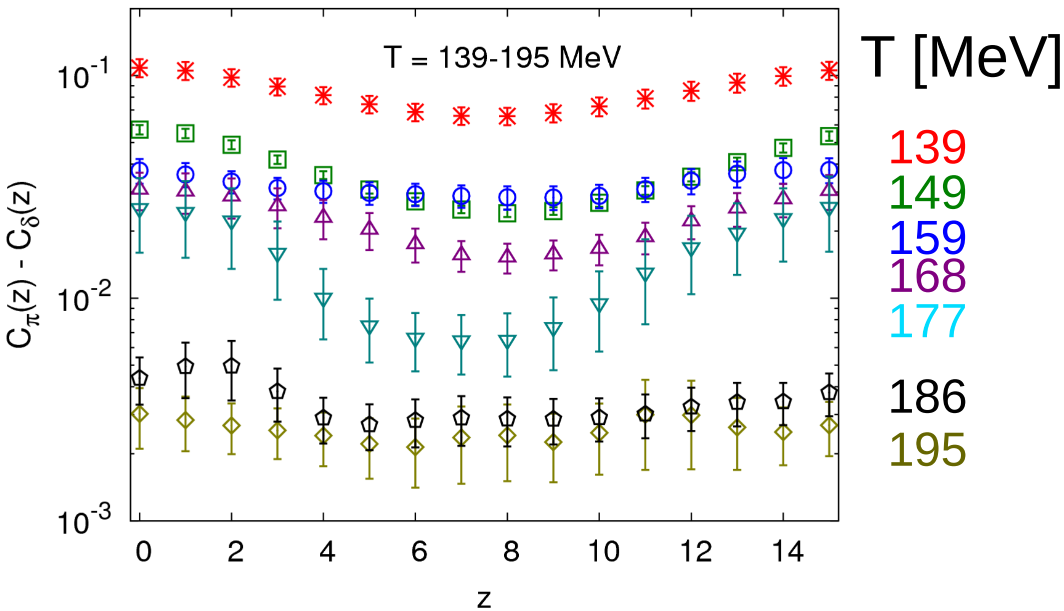
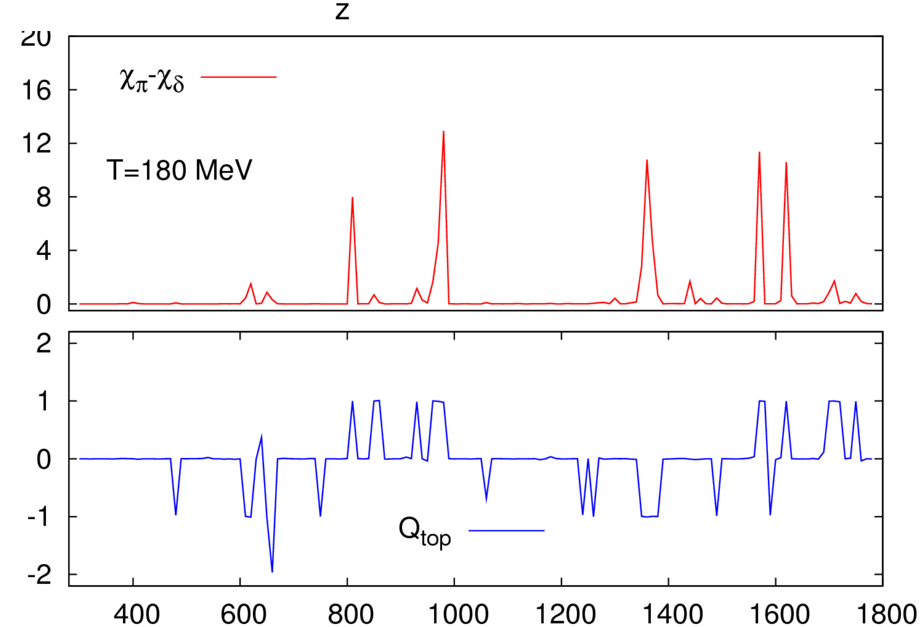


non-zero differences are generated on configurations with non-zero topology



thermodynamics with domain wall fermions

hotQCD, arXiv:0512.xxxx

Conclusions

– the "crossover transition" in QCD is sensitive to universal scaling properties of the second order phase transition in the chiral limit

– in the chiral limit taken at physical values of the strange quark mass the transition seems to be second order, belonging to the 3d, O(4) universality class; UA(1) remains broken at Tc

– the transition temperature and the freeze-out temperature agree within current statistical accuracy at zero and non-zero baryon chemical potential at least down to beam energies of 20GeV

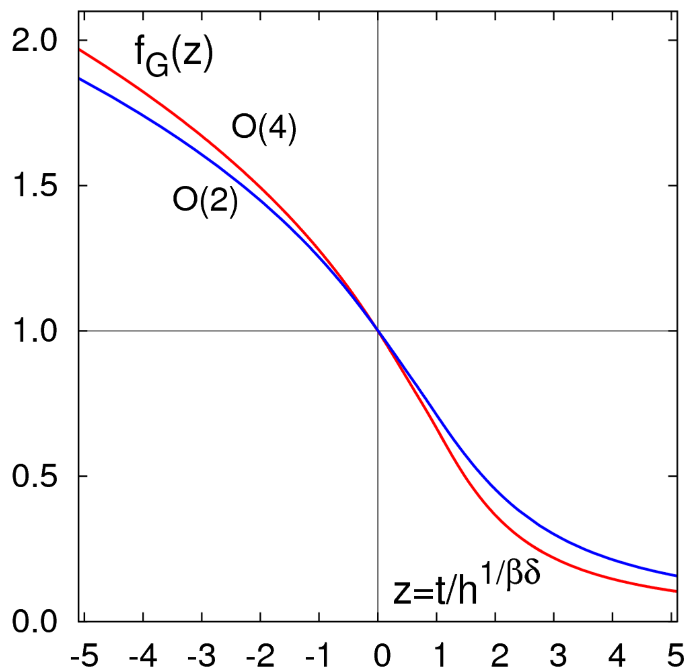
O(N) scaling and the chiral transition

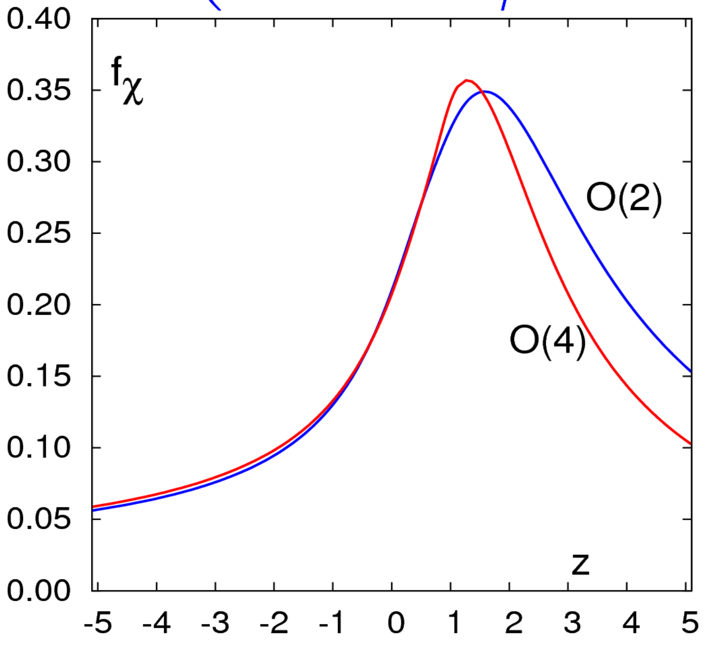
- In the vicinity of (t,h)=(0,0) the chiral order parameter and its susceptibility are given in terms of scaling functions

$$M = h^{1/\delta} f_G(z) \quad , \quad \chi_M = \partial M/\partial h = h^{1/\delta - 1} f_\chi(z)$$

$$\chi_t = \partial M/\partial T = \frac{1}{t_0 T_c} h^{(\beta-1)/\delta\beta} f'_G(z)$$

$$f_\chi(z) = \frac{1}{\delta}\left(f_G(z) - \frac{z}{\beta} f'_G(z) \right)$$

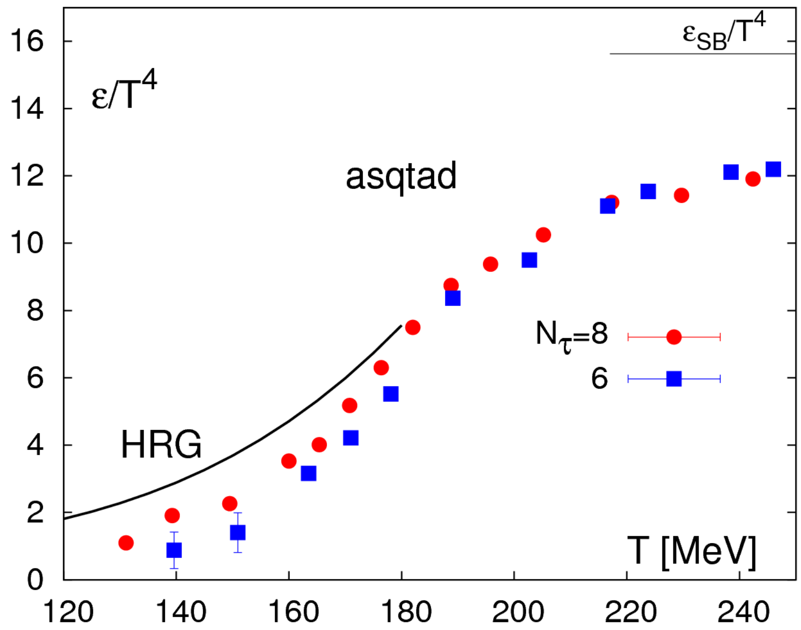


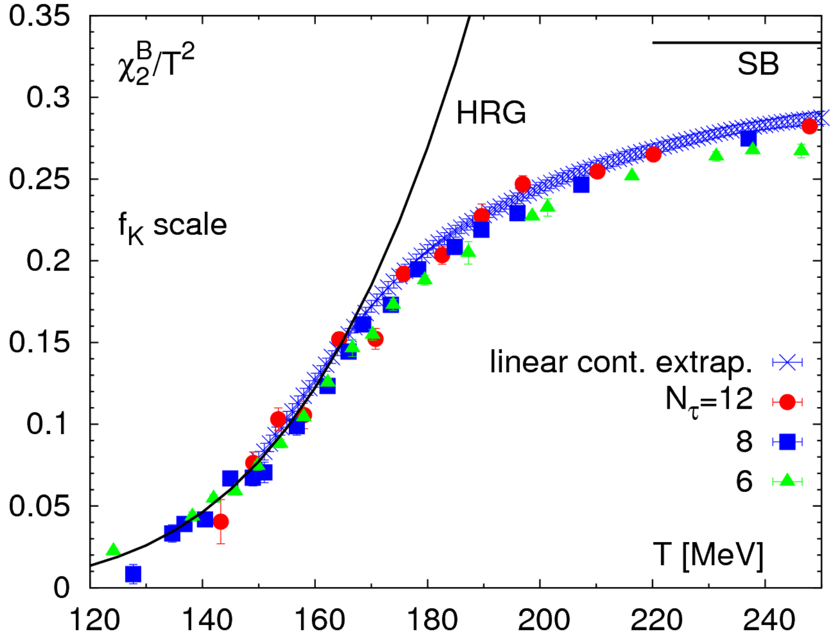


known from 3d O(N) spin model

J. Engels et al., 2001/2003

Energy density vs. quark number susceptibility





$$\frac{\epsilon}{T^4} = \frac{1}{VT^3}\frac{\partial \ln Z}{\partial T}$$

$$\sim -h^{(1-\alpha)} f_s'(z)$$

$$+\left.\frac{\partial f_r(T,V,\vec{\mu})}{\partial T}\right|_{\vec{\mu}=0}$$

$$\chi_2^B = \frac{1}{VT^3}\left.\frac{\partial^2 \ln Z}{\partial(\mu_B/T)^2}\right|_{\mu_B=0}$$

$$\sim -h^{(1-\alpha)} f_s'(z)$$

$$+\left.\frac{\partial^2 f_r(T,V,\vec{\mu})}{\partial(\mu_B/T)^2}\right|_{\vec{\mu}=0}$$

The chiral phase transition

$$U_V(1) \times U_A(1) \times SU_L(n_f) \times SU_R(n_f)$$

$$\bar\psi \mathcal{M} \psi \sim \bar\psi_L \not{D}_\mu \psi_L + \bar\psi_R \not{D}_\mu \psi_R - m_q (\bar\psi_L \psi_R + \bar\psi_R \psi_L)$$

chiral projection: $\psi = \psi_L + \psi_R$

$$P_\epsilon = \frac{1}{2}(1 + \epsilon\gamma_5) \;,\; \epsilon = \pm 1 \;, \quad P_\epsilon^2 = P_\epsilon \;,\; P_+ P_- = 0$$

$$\psi_L = P_+ \psi \quad , \quad \psi_R = P_- \psi$$

$$\bar\psi_L = \bar\psi P_- \quad , \quad \bar\psi_R = \bar\psi P_+$$

$$\bar\psi\psi = \bar\psi_L \psi_R + \bar\psi_R \psi_L$$

mass term breaks left-right symmetry $\Longrightarrow$ order parameter